INTERNATIONAL ACADEMY
OF EDUCATION

INTERNATIONAL BUREAU
OF EDUCATION

Improving student achievement in mathematics

By Douglas A. Grouws
and Kristin J. Cebulla

EDUCATIONAL PRACTICES SERIES–4

The International Academy of Education

The International Academy of Education (IAE) is a not-for-profit scientific association that promotes educational research, its dissemination, and the implementation of its implications. Founded in 1986, the Academy is dedicated to strengthening the contributions of research, solving critical educational problems throughout the world, and providing better communication among policy makers, researchers and practitioners. The seat of the Academy is at the Royal Academy of Science, Literature and Arts in Brussels, Belgium, and its co-ordinating centre is at Curtin University of Technology in Perth, Australia.

The general aim of the IAE is to foster scholarly excellence in all fields of education. Towards this end, the Academy provides timely syntheses of research-based evidence of international importance. The Academy also provides critiques of research, its evidentiary basis, and its application to policy.

The current members of the Board of Directors of the Academy are:

- Erik De Corte, University of Leuven, Belgium *(President)*;
- Herbert Walberg, University of Illinois at Chicago, United States of America *(Vice-President)*;
- Barry Fraser, Curtin University of Technology, Australia *(Executive Director)*;
- Jacques Hallak, UNESCO, Paris, France;
- Michael Kirst, Stanford University, United States of America;
- Ulrich Teichler, University of Kassel, Germany;
- Margaret Wang, Temple University, United States of America.

Preface

This booklet has been adapted for inclusion in the Educational Practices Series developed by the International Academy of Education (IAE) and distributed by the International Bureau of Education (IBE) and the Academy. As part of its mission, the Academy provides timely syntheses of research on educational topics of international importance. This booklet is the fourth in the series on educational practices that generally improve learning.

The material was originally prepared for the *Handbook of research on improving student achievement,* edited by Gordon Cawelti and published in a second edition in 1999 by the Educational Research Service (ERS). The Handbook, which also includes chapters on subjects such as generic practices and science, is available from ERS (2000 Clarendon Boulevard, Arlington, VA 22201-2908, United States of America; phone (1) 800-791-9308; fax (1) 800-791-9309; and website: www.ers.org).

ERS is a not-for-profit research foundation serving the research and information needs of educational leaders and the public. Established in 1973, ERS is sponsored by seven organizations: the American Association of School Administrators; the American Association of School Personnel Administrators; the Association of School Business Officials; the Council of Chief State School Officers; the National Association of Elementary School Principals; the National Association of Secondary School Principals; and the National School Public Relations Association. As Vice-President of the Academy and editor of the present series, I thank ERS officials for allowing the IAE and the IBE to make available the material adapted from the *Handbook* to educators around the world.

The first author of the present pamphlet, Douglas A. Grouws, is Professor of Mathematics Education at the University of Iowa. He was the editor of the *Handbook of research on mathematics teaching and learning* (Macmillan, 1992) and has a large number of other publications on research in mathematics education to his credit. He has made invited research presentations in Australia, China, Hungary, Guam, India, Japan, Mexico, Thailand and the United Kingdom. He has directed several research projects for the National Science Foundation (NSF) and other agencies in the areas of mathematical problem-solving and classroom teaching practices. His current NSF work involves mathematics and technology. He received his Ph.D. from the University of Wisconsin.

The second author, Kristin J. Cebulla, is a mathematics education doctoral student at the University of Iowa. She previously taught middle-school mathematics. Before teaching, she worked as a research chemical engineer. She received her bachelor of science degree in mathematics and chemical engineering from the University of Notre Dame and her master's degree from the University of Mississippi.

The principles described in this booklet are derived in large part from the United States and other English-speaking countries. Other research also has important implications for the teaching of mathematics. An example is Realistic Mathematics Education, initiated by H. Freudenthal and developed since the early 1970s at the University of Utrecht (Dordrecht, The Netherlands, Kluwer, 1991). Another example is research on problem-solving summarized in a recent book by L. Verschaffel, B. Greer, and E. De Corte—*Making sense of word problems* (Lisse, The Netherlands, Swets & Zeitlinger, 2000).

The officers of the International Academy of Education are aware that this booklet is based on research carried out primarily in economically advanced countries. The booklet, however, focuses on aspects of learning that appear to be universal in much formal schooling. The practices seem likely to be generally applicable throughout the world. Even so, the principles should be assessed with reference to local conditions, and adapted accordingly. In any educational setting, suggestions or guidelines for practice require sensitive and sensible application and continuing evaluation.

HERBERT J. WALBERG
Editor, IAE Educational Practices Series
University of Illinois at Chicago

Table of contents

This publication has been produced in 2000 by the International Academy of Education (IAE), Palais des Académies, 1, rue Ducale, 1000 Brussels, Belgium, and the International Bureau of Education (IBE), P.O. Box 199, 1211 Geneva 20, Switzerland.

Printed in Switzerland by PCL, Lausanne.

Introduction

This booklet summarizes the mathematics chapter from the *Handbook of research on improving student achievement,* second edition, published by the Educational Research Service. The *Handbook* is based on the idea that, in order to succeed, efforts to improve instruction must focus on the existing knowledge base in respect of effective teaching and learning. The *Handbook* was specifically designed to help school administrators and teachers carry out their evolving instructional leadership roles by giving them a ready source of authoritative yet practitioner-based information about research on effective teaching and learning.

The practices identified in this booklet reflect a mixture of emerging strategies and practices in long-term use. The authors briefly summarize the research supporting each practice, describe how this research might be applied in actual classroom practice, and list the most important studies that support the practice. A complete list of references is provided at the end of the booklet for readers who want to study and understand the practices more fully.

In most cases, the results of research on specific teaching practices show only small or moderate gains. In education, we need to understand, carefully select, and use combinations of teaching practices that together increase the probability of helping students learn, knowing that these practices may not work in all classrooms at all times.

The strongest possibility of improving student learning emerges where schools implement multiple changes in the teaching and learning activities affecting the daily life of students. For example, if the aim is to improve students' scientific problem-solving skills, the school might plan to introduce training for teachers in (1) use of the learning cycle approach; (2) use of computer simulations; and (3) systemic approaches to problem solving. To simultaneously plan for the training and other provisions needed to sustain all three of these changes would be no small undertaking, but would hold great promise for improving the quality of student problem-solving.

The research findings presented in this booklet provide a starting point for developing comprehensive school plans to improve mathematics instruction. Teachers and school leaders will inevitably need time for further study, discussion and other exposure to what a particular practice entails before deciding to include it in their school's plans.

The complexities involved in putting the knowledge base on improving student achievement to work in classrooms must be recognized. As Dennis Sparks writes in his chapter on staff development in the *Handbook of research on improving student achievement,* schools and school districts have a responsibility to establish a culture in which teachers can exercice their professional competence, explore promising practices and share information among themselves, while keeping the focus on the ultimate goal of staff development—the improvement of student learning.

Improving teacher effectiveness

The number of research studies conducted in mathematics education over the past three decades has increased dramatically (Kilpatrick, 1992). The resulting research base spans a broad range of content, grade levels and research methodologies. The results from these studies, together with relevant findings from research in other domains, such as cognitive psychology, are used to identify the successful teaching strategies and practices.

Teaching and learning mathematics are complex tasks. The effect on student learning of changing a single teaching practice may be difficult to discern because of the simultaneous effects of both the other teaching activities that surround it and the context in which the teaching takes place.

Thus, as teachers seek to improve their teaching effectiveness by changing their instructional practices, they should carefully consider the teaching context, giving special consideration to the types of students they teach. And, further, they should not judge the results of their new practices too quickly. Judgements about the appropriateness of their decisions must be based on more than a single outcome. If the results are not completely satisfactory, teachers should consider the circumstances that may be diminishing the impact of the practices they are implementing. For example, the value of a teacher focusing more attention on teaching for meaning may not be demonstrated if student assessments concentrate on rote recall of facts and proficient use of isolated skills.

The quality of the implementation of a teaching practice also greatly influences its impact on student learning. The value of using manipulative materials to investigate a concept, for example, depends not only on *whether* manipulatives are used, but also on *how* they are used with the students. Similarly, small-group instruction will benefit students only if the teacher knows when and how to use this teaching practice. Hence, as a teacher implements any of the recommendations, it is essential that he or she constantly monitors and adjusts the way the practice is implemented in order to optimize improvements in quality.

These cautions notwithstanding, the research findings indicate that certain teaching strategies and methods are worth careful consideration as teachers strive to improve their mathematics teaching practices. As readers examine the suggestions that follow, it will become clear that many of the practices are interrelated. There is also considerable variety in the practices that have been found to be effective, and so most teachers should be able to identify ideas they would like to try in their classrooms. The practices are not mutually exclusive; indeed, they tend to be complementary. The logical consistency and variety in the suggestions from research make them both interesting and practical.

The authors wish to acknowledge the following colleagues who made helpful suggestions: Tom Cooney, Professor of Mathematics, University of Georgia; James Hiebert, Professor of Mathematics Education, University of Delaware; Judy Sowder, Professor of Mathematics, San Diego State University; and Terry Wood, Professor of Mathematics Education, Purdue University.

1. Opportunity to learn

The extent of the students' opportunity to learn mathematics content bears directly and decisively on student mathematics achievement.

Research findings

The term 'opportunity to learn' (OTL) refers to what is studied or embodied in the tasks that students perform. In mathematics, OTL includes the scope of the mathematics presented, how the mathematics is taught, and the match between students' entry skills and new material.

The strong relationship between OTL and student performance in mathematics has been documented in many research studies. The concept was studied in the First International Mathematics Study (Husén), where teachers were asked to rate the extent of student exposure to particular mathematical concepts and skills. Strong correlations were found between student OTL scores and mean student achievement scores in mathematics, with high OTL scores associated with high achievement. The link between student mathematics achievement and opportunity to learn was also found in subsequent international studies, such as the Second International Mathematics Study (McKnight et al.) and the Third International Mathematics and Science Study (TIMSS) (Schmidt, McKnight & Raizen).

As might be expected, there is also a positive relationship between total time allocated to mathematics and general mathematics achievement. Suarez et al., in a review of research on instructional time, found strong support for the link between allocated instructional time and student performance. Internationally, Keeves found a significant relationship across Australian states between achievement in mathematics and total curriculum time spent on mathematics.

In spite of these research findings, many students still spend only minimal amounts of time in the mathematics class. For instance, Grouws and Smith, in an analysis of data from the 1996 National Assessment of Educational Progress (NAEP) mathematics study, found that 20% of eighth-grade students had thirty minutes or less for mathematics instruction each day.

Research has also found a strong relationship between mathematics-course taking at the secondary school level and student achievement. Reports from the NAEP in mathematics showed that 'the number of advanced mathematics courses taken was the most powerful predictor of students' mathematics performance after adjusting for variations in home background'.

Textbooks are also related to student OTL, because many textbooks do not contain much content that is new to students. The lack of attention to new material and heavy emphasis on review in many textbooks are of particular concern at the elementary school and middle-school levels. Flanders examined several textbook series and found that fewer than 50% of the pages in textbooks for grades two through eight contained any material new to students. In a review of a dozen middle-grade mathematics textbook series, Kulm, Morris and Grier found that most traditional textbook series lack many of the content recommendations made in recent standards documents.

United States data from TIMSS showed important differences in the content taught to students in different mathematics classes or tracks. For example, students in remedial classes, typical eighth-grade classes and pre-algebra classes were exposed to very different mathematics contents, and their achievement levels varied accordingly. The achievement tests used in international studies and in NAEP assessments measure important mathematical outcomes and have commonly provided a broad and representative coverage of mathematics. Moreover, the tests have generally served to measure what even the most able students know and do not know. Consequently, they provide reasonable outcome measures for research that examines the importance of opportunity to learn as a factor in student mathematics achievement.

In the classroom

The findings about the relationship between opportunity to learn and student achievement have important implications for teachers. In particular, it seems prudent to allocate sufficient time for mathematics instruction at every grade level. Short class periods in mathematics, instituted for whatever practical or philosophical reason, should be seriously questioned. Of special concern are the 30–35 minute class periods for mathematics being implemented in some middle schools.

Textbooks that devote major attention to review and that address little new content each year should be avoided, or their use should be heavily supplemented in appropriate ways. Teachers should use textbooks as just one instructional tool among many, rather than feel duty-bound to go through the textbook on a one-section-per-day basis.

Teachers must ensure that students are given the opportunity to learn important content and skills. If students are to compete effectively in a global, technologically oriented society, they must be taught the mathematical skills needed to do so. Thus, if problem solving is essential, explicit attention must be given to it on a regular and sustained basis. If we expect students to develop number sense, it is important to attend to mental computation and estimation as part of the curriculum. If proportional reasoning and deductive reasoning are important, attention must be given to them in the curriculum implemented in the classroom.

It is important to note that opportunity to learn is related to equity issues. Some educational practices differentially affect particular groups of students' opportunity to learn. For example, a recent American Association of University Women study showed that boys' and girls' use of technology is markedly different. Girls take fewer computer science and computer design courses than do boys. Furthermore, boys often use computers to programme and solve problems, whereas girls tend to use the computer primarily as a word processor. This suggests that, as technology is used in the mathematics classroom, teachers must assign tasks and responsibilities to students in such a way that both boys and girls have active learning experiences with the technological tools employed.

OTL is also affected when low-achieving students are tracked into special 'basic skills' curricula, oriented towards developing procedural skills, with little opportunity to develop problem solving and higher-order thinking abilities. The impoverished curriculum frequently provided to these students is an especially serious problem because the ideas and concepts frequently untaught or de-emphasized are the very ones needed in everyday life and in the workplace.

References: American Association of University Women, 1998; Atanda, 1999; Flanders, 1987; Grouws & Smith, in press; Hawkins, Stancavage & Dossey, 1998; Husén, 1967; Keeves, 1976, 1994; Kulm, Morris & Grier, 1999; McKnight et al., 1987; Mullis, Jenkins & Johnson, 1994; National Center for Education Statistics, 1996, 1997, 1998; Schmidt, McKnight & Raizen, 1997; Secada, 1992; Suarez et al., 1991.

2. Focus on meaning

Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.

Research findings

There is a long history of research, going back to the 1940s and the work of William Brownell, on the effects of teaching for meaning and understanding in mathematics. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations. These results have also been found in studies conducted in high-poverty areas.

In the classroom

As might be expected, the concept of 'teaching for meaning' has varied somewhat from study to study, and has evolved over time. Teachers will want to consider how various interpretations of this concept can be incorporated into their classroom practice.

- *Emphasize the mathematical meanings of ideas, including how the idea, concept or skill is connected in multiple ways to other mathematical ideas in a logically consistent and sensible manner.* Thus, for subtraction, emphasize the inverse, or 'undoing', relationship between it and addition. In general, emphasis on meaning was common in early research in this area in the late 1930s, and its purpose was to avoid the mathematical meaningfulness of the ideas taught receiving only minor attention compared to a heavy emphasis on the social uses and utility of mathematics in everyday life.
- *Create a classroom learning context in which students can construct meaning.* Students can learn important mathematics both in contexts that are closely connected to real-life situations and in those that are purely mathematical. The abstractness of a learning environment and how students relate to it must be carefully regulated, closely monitored and thoughtfully chosen. Consideration should be given to students' interests and backgrounds. The mathematics

taught and learned must seem reasonable to students and make sense to them. An important factor in teaching for meaning is connecting the new ideas and skills to students' past knowledge and experience.

- *Make explicit the connections between mathematics and other subjects.* For example, instruction could relate data-gathering and data-representation skills to public opinion polling in social studies. Or, it could relate the mathematical concept of direct variation to the concept of force in physics to help establish a real-world referent for the idea.
- *Attend to student meanings and student understanding in instruction.* Students' conceptions of the same idea will vary, as will their methods of solving problems and carrying out procedures. Teachers should build on students' intuitive notions and methods in designing and implementing instruction.

References: Aubrey, 1997; Brownell, 1945, 1947; Carpenter et al., 1998; Cobb et al., 1991; Fuson, 1992; Good, Grouws & Ebmeier, 1983; Hiebert & Carpenter, 1992; Hiebert & Wearne, 1996; Hiebert et al., 1997; Kamii, 1985, 1989, 1994; Knapp, Shields & Turnbull, 1995; Koehler & Grouws, 1992; Skemp, 1978; Van Engen, 1949; Wood & Sellers, 1996, 1997.

3. Learning new concepts and skills while solving problems

Students can learn both concepts and skills by solving problems.

Research findings

Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later. Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students without conceptual understanding are able to acquire procedural knowledge when the skill is taught, but research suggests that students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge.

Research by Heid suggests that students are able to understand concepts without prior or concurrent skill development. In her research with calculus students, instruction was focused almost entirely on conceptual understanding. Skills were taught briefly at the end of the course. On procedural skills, the students in the conceptual-understanding approach performed as well as those taught with a traditional approach. Furthermore, these students significantly outperformed traditional students on conceptual understanding.

Mack demonstrated that students' rote (and frequently faulty) knowledge often interferes with their informal (and usually correct) knowledge about fractions. She successfully used students' informal knowledge to help them understand symbols for fractions and develop algorithms for operations. Fawcett's research with geometry students suggests that students can learn basic concepts, skills and the structure of geometry through problem solving.

In the classroom

There is evidence that students can learn new skills and concepts while they are working out solutions to problems. For example, armed with only a knowledge of basic addition, students can extend their learning by developing informal algorithms for addition of larger numbers. Similarly, by solving carefully chosen non-routine problems, students can develop an understanding of many important mathematical ideas, such as prime numbers and perimeter/area relations.

Development of more sophisticated mathematical skills can also be approached by treating their development as a problem for students to solve. Teachers can use students' informal and intuitive knowledge in other areas to develop other useful procedures. Instruction can begin with an example for which students intuitively know the answer. From there, students are allowed to explore and develop their own algorithm. For instance, most students understand that starting with four pizzas and then eating a half of one pizza will leave three and a half pizzas. Teachers can use this knowledge to help students develop an understanding of subtraction of fractions.

Research suggests that it is not necessary for teachers to focus first on skill development and then move on to problem solving. Both can be done together. Skills can be developed on an as-needed basis, or their development can be supplemented through the use of technology. In fact, there is evidence that if students are initially drilled too much on isolated skills, they have a harder time making sense of them later.

References: Cognition and Technology Group, 1997; Fawcett, 1938; Heid, 1988; Hiebert & Wearne, 1996; Mack, 1990; Resnick & Omanson, 1987; Wearne & Hiebert, 1988.

4. Opportunities for both invention and practice

Giving students both an opportunity to discover and invent new knowledge and an opportunity to practise what they have learned improves student achievement

Research findings

Data from the TIMSS video study show that over 90% of mathematics class time in United States eighth-grade classrooms is spent practising routine procedures, with the remainder of the time generally spent applying procedures in new situations. Virtually no time is spent inventing new procedures and analysing unfamiliar situations. In contrast, students at the same grade level in typical Japanese classrooms spend approximately 40% of instructional time practising routine procedures, 15% applying procedures in new situations, and 45% inventing new procedures and analysing new situations.

Research evidence suggests that students need opportunities for both practice and invention. The findings from a number of research studies show that when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas.

Many successful reform-oriented programmes include time for students to practise what they have learned and discovered. Students need opportunities to practise what they are learning and to experience performing the kinds of tasks in which they are expected to demonstrate competence. For example, if teachers want students to be proficient in problem solving, students must be given opportunities to practise problem solving. If strong deductive reasoning is a goal, student work must include tasks that require such reasoning. And, of course, if competence in procedures is an objective, the curriculum must include attention to such procedures.

In the classroom

Clearly, a balance is needed between the time students spend practising routine procedures and the time which they devote to inventing and discovering new ideas. Teachers need not choose between these activities; indeed, they must not make a choice if students are to develop the mathematical power they need. Teachers must strive to ensure that both activities are included in appropriate proportions and in appropriate ways. The research cited above suggests that attention to them is currently out of balance and that too frequently there is an over-emphasis on skill work, with few opportunities for students to engage in sense-making and discovery-oriented activities.

To increase opportunities for invention, teachers should frequently use non-routine problems, periodically introduce a lesson involving a new skill by posing it as a problem to be solved, and regularly allow students to build new knowledge based on their intuitive knowledge and informal procedures.

References: Boaler, 1998; Brownell, 1945, 1947; Carpenter et al., 1998; Cobb et al., 1991; Cognition and Technology Group, 1997; Resnick, 1980; Stigler & Hiebert, 1997; Wood & Sellers, 1996, 1997.

5. Openness to student solution methods and student interaction

Teaching that incorporates students' intuitive solution methods can increase student learning, especially when combined with opportunities for student interaction and discussion.

Research findings

Recent results from the TIMSS video study have shown that Japanese classrooms use student solution methods extensively during instruction. Interestingly, the same teaching technique appears in many successful American research projects. Findings from American studies clearly demonstrate two important principles that are associated with the development of students' deep conceptual understanding of mathematics. First, student achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics. These results have been clearly demonstrated in the primary grades and are beginning to be shown at higher-grade levels.

Second, structuring instruction around carefully chosen problems, allowing students to interact when solving these problems, and then providing opportunities for them to share their solution methods result in increased achievement on problem-solving measures. Importantly, these gains come without a loss of achievement in the skills and concepts measured on standardized achievement tests.

Research has also demonstrated that when students have opportunities to develop their own solution methods, they are better able to apply mathematical knowledge in new problem situations.

In the classroom

Research results suggest that teachers should concentrate on providing opportunities for students to interact in problem-rich situations. Besides providing appropriate problem-rich situations, teachers must encourage students to find their own solution methods and give them opportunities to share and compare their solution methods and answers. One way to organize such instruction is to have students work in small groups initially and then share ideas and solutions in a whole-class discussion.

One useful teaching technique is for teachers to assign an interesting problem for students to solve and then move about the room as they work, keeping track of which students are using which strategies (taking notes if necessary). In a whole-class setting, the teacher can then call on students to discuss their solution methods in a pre-determined and carefully considered order, these methods often ranging from the most basic to more formal or sophisticated ones. This teaching structure is used successfully in many Japanese mathematics lessons.

References: Boaler, 1998; Carpenter et al., 1988, 1989, 1998; Cobb, Yackel & Wood, 1992; Cobb et al., 1991; Cognition and Technology Group, 1997; Fennema, Carpenter & Peterson, 1989; Fennema et al., 1993, 1996; Hiebert & Wearne, 1993, 1996; Kamii, 1985, 1989, 1994; Stigler & Hiebert, 1997; Stigler et al., 1999; Wood, Cobb & Yackel, 1995; Wood et al., 1993; Yackel, Cobb & Wood, 1991.

6. Small-group learning

Using small groups of students to work on activities, problems and assignments can increase student mathematics achievement.

Research findings

Considerable research evidence within mathematics education indicates that using small groups of various types for different classroom tasks has positive effects on student learning. Davidson, for example, reviewed almost eighty studies in mathematics that compared student achievement in small-group settings with traditional whole-class instruction. In more than 40% of these studies, students in the classes using small-group approaches significantly outscored control students on measures of student performance. In only two of the seventy-nine studies did control-group students perform better than the small-group students, and in these studies there were some design irregularities.

From a review of ninety-nine studies of co-operative group-learning methods at the elementary and secondary school levels, Slavin concluded that co-operative methods were effective in improving student achievement. The most effective methods emphasized both group goals and individual accountability.

From a review by Webb of studies examining peer interaction and achievement in small groups (seventeen studies, grades 2–11), several consistent findings emerged. First, giving an explanation of an idea, method or solution to a team mate in a group situation was positively related to achievement. Second, receiving 'non-responsive' feedback (no feedback or feedback that is not pertinent to what one has said or done) from team mates was negatively related to achievement. Webb's review also showed that group work was most effective when students were taught how to work in groups and how to give and receive help. Received help was most effective when it was in the form of elaborated explanations (not just the answer) and then applied by the student either to the current problem or to a new problem.

Qualitative investigations have shown that other important and often unmeasured outcomes beyond improved general achievement can result from small-group work. In one such investigation, Yackel, Cobb and Wood studied a second-grade classroom in which small-group problem solving followed by whole-class discussion was the primary instructional strategy for the entire school year. They found that this approach created many learning opportunities that do not typically occur in traditional classrooms, including opportunities for collaborative dialogue and resolution of conflicting points of view.

Slavin's research showed positive effects of small-group work on cross-ethnic relations and student attitudes towards school.

In the classroom

Research findings clearly support the use of small groups as part of mathematics instruction. This approach can result in increased student learning as measured by traditional achievement measures, as well as in other important outcomes.

When using small groups for mathematics instruction, teachers should:

- choose tasks that deal with important mathematical concepts and ideas;
- select tasks that are appropriate for group work;
- consider having students initially work individually on a task and then follow this with group work where students share and build on their individual ideas and work;
- give clear instructions to the groups and set clear expectations for each;
- emphasize both group goals and individual accountability;
- choose tasks that students find interesting;
- ensure that there is closure to the group work, where key ideas and methods are brought to the surface either by the teacher or the students, or both.

Finally, as several research studies have shown, teachers should not think of small groups as something that must always be used or never be used. Rather, small-group instruction should be thought of as an instructional practice that is appropriate for certain learning objectives, and as a practice that can work well with other organizational arrangements, including whole-class instruction.

References: Cohen, 1994; Davidson, 1985; Laborde, 1994; Slavin, 1990, 1995; Webb, 1991; Webb, Troper & Fall, 1995; Yackel, Cobb & Wood, 1991.

7. Whole-class discussion

Whole-class discussion following individual and group work improves student achievement.

Research findings

Research suggests that whole-class discussion can be effective when it is used for sharing and explaining the variety of solutions by which individual students have solved problems. It allows students to see the many ways of examining a situation and the variety of appropriate and acceptable solutions.

Wood found that whole-class discussion works best when discussion expectations are clearly understood. Students should be expected to evaluate each other's ideas and reasoning in ways that are not critical of the sharer. This helps to create an environment in which students feel comfortable sharing ideas and discussing each other's methods and reasoning. Furthermore, students should be expected to be active listeners who participate in the discussion and feel a sense of responsibility for each other's understanding.

Cognitive research suggests that conceptual change and progression of thought result from the mental processes involved in the resolution of conflict and contradiction. Thus, confusion and conflict during whole-class discussion have considerable potential for increasing student learning when carefully managed by the teacher. As students address challenges to their methods, they strengthen their understanding of concepts and procedures by working together to resolve differences in thinking or confusions in reasoning. In a sense, the discussion becomes a collaborative problem-solving effort. Each individual then is contributing to the total outcome of the problem-solving situation. This discussion helps produce the notion of commonly held knowledge (public knowledge).

In the classroom

It is important that whole-class discussion follow student work on problem-solving activities. The discussion should be a summary of individual work in which key ideas are brought to the surface. This can be accomplished through students presenting and discussing their individual solution methods, or through other methods of achieving closure that are led by the teacher, the students, or both.

Whole-class discussion can also be an effective diagnostic tool for determining the depth of student understanding and identifying misconceptions. Teachers can identify areas of difficulty for particular students, as well as ascertain areas of student success or progress.

Whole-class discussion can be an effective and useful instructional practice. Some of the instructional opportunities offered in whole-class discussion do not occur in small group or individual settings. Thus, whole-class discussion has an important place in the classroom together with other instructional practices.

References: Ball, 1993; Cobb et al., 1992; Wood, 1999.

8. Number sense

Teaching mathematics with a focus on number sense encourages students to become problem solvers in a wide variety of situations and to view mathematics as a discipline in which thinking is important.

Research findings

'Number sense' relates to having an intuitive feel for number size and combinations, as well as the ability to work flexibly with numbers in problem situations in order to make sound decisions and reasonable judgements. It involves being able to use flexibly the processes of mentally computing, estimating, sensing number magnitudes, moving between representation systems for numbers, and judging the reasonableness of numerical results.

Markovits and Sowder studied seventh-grade classrooms where special units on number magnitude, mental computation and computational estimation were taught. From individual interviews, they determined that after this special instruction students were more likely to use strategies that reflected sound number sense, and that this was a long-lasting change.

Other important research in this area involves the integration of the development of number sense with the teaching of other mathematical topics, as opposed to teaching separate lessons on aspects of number sense. In a study of second graders, Cobb and his colleagues found that students' number sense was improved as a result of a problem-centred curriculum that emphasized student interaction and self-generated solution methods. Almost every student developed a variety of strategies to solve a wide range of problems. Students also demonstrated other desirable affective outcomes, such as increased persistence in solving problems.

Kamii worked with primary-grade teachers as they attempted to implement an instructional approach rooted in a constructivist theory of learning that is based on the work of Piaget. Central to the instructional approach was providing situations for students to develop their own meanings, methods and number sense. Data obtained from interviews with students showed that the treatment group demonstrated a greater autonomy, conceptual understanding of place value, and ability to do estimation and mental computation than did students in comparison classrooms.

In the classroom

Attention to number sense when teaching a wide variety of mathematical topics tends to enhance the depth of student ability in this area. Competence in the many aspects of number sense is an important mathematical outcome for students. Over 90% of the computation done outside the classroom is done without pencil and paper, using mental computation, estimation or a calculator. However, in many classrooms, efforts to instil number sense are given insufficient attention.

As teachers develop strategies to teach number sense, they should strongly consider moving beyond a unit-skills approach (i.e. a focus on single skills in isolation) to a more integrated approach that encourages the development of number sense in all classroom activities, from the development of computational procedures to mathematical problem solving. Although more research is needed, an integrated approach to number sense will be likely to result not only in greater number sense but also in other equally important outcomes.

References: Cobb et al., 1991; Greeno, 1991; Kamii, 1985, 1989, 1994; Markovits & Sowder, 1994; Reys & Barger, 1994; Reys et al., 1991; Sowder, 1992*a*, 1992*b*.

9. Concrete materials

Long-term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes towards mathematics.

Research findings

Many studies show that the use of concrete materials can produce meaningful use of notational systems and increase student concept development. In a comprehensive review of activity-based learning in mathematics in kindergarten through grade eight, Suydam and Higgins concluded that using manipulative materials produces greater achievement gains than not using them. In a more recent meta-analysis of sixty studies (kindergarten through post-secondary) that compared the effects of using concrete materials with the effects of more abstract instruction, Sowell concluded that the long-term use of concrete instructional materials by teachers knowledgeable in their use improved student achievement and attitudes.

In spite of generally positive results, there are some inconsistencies in the research findings. As Thompson points out, the research results concerning concrete materials vary, even among treatments that were closely controlled and monitored and that involved the same concrete materials. For example, in studies by Resnick and Omanson and by Labinowicz, the use of base-ten blocks showed little impact on children's learning. In contrast, both Fuson and Briars and Hiebert and Wearne reported positive results from the use of base-ten blocks.

The differences in results among these studies might be due to the nature of the students' engagement with the concrete materials and their orientation towards the materials in relation to notation and numerical values. They might also be due to different orientations in the studies, with regard to the role of computational algorithms and how they should be developed in the classroom. In general, however, the ambiguities in some of the research findings do not undermine the general consensus that concrete materials are valuable instructional tools.

In the classroom

Although successful teaching requires teachers to carefully choose their procedures on the basis of the context in which they will be used, available research suggests that teachers should use manipulative materials in mathematics instruction more regularly in order to give students hands-on experience that helps them construct useful meanings for the mathematical ideas they are learning. Use of the same material to teach multiple ideas over the course of schooling has the advantage of shortening the amount of time it takes to introduce the material and also helps students to see connections between ideas.

The use of concrete material should not be limited to demonstrations. It is essential that children use materials in meaningful ways rather than in a rigid and prescribed way that focuses on remembering rather than on thinking. Thus, as Thompson says, 'before students can make productive use of concrete materials, they must first be committed to making sense of their activities and be committed to expressing their sense in meaningful ways. Further, it is important that students come to see the two-way relationship between concrete embodiments of a mathematical concept and the notational system used to represent it.'

References: Fuson & Briars, 1990; Hiebert & Wearne, 1992; Labinowicz, 1985; Leinenbach & Raymond, 1996; Resnick & Omanson, 1987; Sowell, 1989; Suydam & Higgins, 1977; Thompson, 1992; Varelas & Becker, 1997.

10. Students' use of calculators

Using calculators in the learning of mathematics can result in increased achievement and improved student attitudes.

Research findings

The impact of calculator use on student learning has been a popular research area in mathematics education. The many studies conducted have quite consistently shown that thoughtful use of calculators in mathematics classes improves student mathematics achievement and attitudes towards mathematics.

From a meta-analysis of seventy-nine non-graphing calculator studies, Hembree and Dessart concluded that the use of hand-held calculators improved student learning. In particular, they found improvement in students' understanding of arithmetical concepts and in their problem-solving skills. Their analysis also showed that students using calculators tended to have better attitudes towards mathematics and much better self-concepts in mathematics than their counterparts who did not use calculators. They also found that there was no loss in student ability to perform paper-and-pencil computational skills when calculators were used as part of mathematics instruction.

Research on the use of scientific calculators with graphing capabilities has also shown positive effects on student achievement. Most studies have found positive effects on students' graphing ability, conceptual understanding of graphs and their ability to relate graphical representations to other representations, such as tables and symbolic representations. Other content areas where improvement has been shown when these calculators have been used in instruction include function concepts and spatial visualization. Other studies have found that students are better problem solvers when using graphing calculators. In addition, students are more flexible in their thinking with regard to solution strategies, have greater perseverance and focus more on trying to understand the problem conceptually rather than simply focusing on computations. However, with increased use of graphing calculators, students are more likely to rely on graphical procedures than on other procedures such as algebraic methods. Most studies of graphing calculators have found no negative effect on basic skills, factual knowledge or computational skills.

In general, research has found that the use of calculators changes the content, methods and skill requirements in mathematics classrooms. Studies have shown that teachers ask more high-level questions when calculators are present, and students become more actively involved through asking questions, conjecturing and exploring when they use calculators.

In the classroom

Research strongly supports the call in *Curriculum and evaluation standards for school mathematics*, published by the National Council of Teachers of Mathematics, for the use of calculators at all levels of mathematics instruction. Using calculators in carefully planned ways can result in increases in student problem-solving ability and improved affective outcomes without a loss in basic skills.

One valuable use for calculators is as a tool for exploration and discovery in problem-solving situations and when introducing new mathematical content. By reducing computation time and providing immediate feedback, calculators help students focus on understanding their work and justifying their methods and results. The graphing calculator is particularly useful in helping to illustrate and develop graphical concepts and in making connections between algebraic and geometric ideas.

In order to accurately reflect their meaningful mathematics performance, students should probably be allowed to use their calculators in achievement tests. Not to do so is a major disruption in many students' usual way of doing mathematics, and an unrealistic restriction because when they are away from the school setting, they will certainly use a calculator in their daily lives and in the workplace. Another factor that argues for calculator use is that students are already permitted to use them in some official tests. Furthermore, some examinations require the candidates to use a graphing calculator.

References: Davis, 1990; Drijvers & Doorman, 1996; Dunham & Dick, 1994; Flores & McLeod, 1990; Giamati, 1991; Groves & Stacey, 1998; Harvey, 1993; Hembree & Dessart, 1986, 1992; Mullis, Jenkins & Johnson, 1994; National Council of Teachers of Mathematics, 1989; Penglase & Arnold, 1996; Rich, 1991; Ruthven, 1990; Slavit, 1996; Smith, 1996; Stacey & Groves, 1994; Wilson & Krapfl, 1994.

Conclusions

This booklet is excerpted from the mathematics chapter of the *Handbook of research on improving student achievement, second edition*. It provides a synthesis of the knowledge base regarding effective practices for improving teaching and learning in mathematics. These materials are intended for use by teachers, principals, other instructional leaders and policy makers who are undertaking the quest to improve student achievement.

The research findings presented are intended to be used as a starting point, which can initiate staff development activities and spark discussion among educators, rather than as a prescription that is equally applicable to all classrooms. As Miriam Met writes in her chapter on foreign languages in the *Handbook of research on improving student achievement*:

Research cannot and does not identify the right or best way to teach […] But research can illuminate which instructional practices are more likely to achieve desired results, with which kinds of learners, and under what conditions. [...] While research may provide direction in many areas, it provides few clear-cut answers in most. Teachers continue to be faced daily with critical decisions about how best to achieve the instructional goals embedded in professional or voluntary state or national standards. A combination of research-suggested instructional practices and professional judgment and experience is most likely to produce [high student achievement].

Thus, this booklet cannot give educators all the information they need to become expert in research-based instructional practices in mathematics. Rather, these materials are designed to be used as a springboard for discussion and further exploration.

For example, one approach to professional development might be to distribute the booklet to teachers, find out which teachers already use certain practices, and then provide opportunities for them to demonstrate the practices to their colleagues. Next, a study group might be formed to pursue further reading and discussion. Both the extensive reference list on page 39 and the list of additional resources on page 35 can serve as a starting point. The study group's work might lay the foundation to plan a staff development programme for the next year or two that would enable teachers to learn more and become confident enough to use the selected practices in their classrooms.

Suggestions from users of the *Handbook*

Since the publication of the first edition of the *Handbook of research on improving student achievement,* the Educational Research Service has asked users how the *Handbook* and related materials have helped them in their efforts to improve instructional practice. Here are a few of their experiences in using these materials for staff development:

- Some teachers suggested reviewing one practice a month through the school year at department meetings. The practice would provide a focus for discussion, with teachers who already used the practice available as resources and as mentors for other teachers who were interested in using the practice in their own classrooms. As one teacher remarked, 'staff development doesn't work when teachers are *told* what they need—often, they then just go along for the ride'.
- One school reported using the materials as a resource when teachers met to discuss alternative approaches that might be used with students who were struggling. The *Handbook* 'provided ideas and was a guide to other resources'.
- Curriculum specialists studied the *Handbook* together, and then met with teachers in their own content areas to review both the contents of the subject-area chapter and the ideas shared among the specialists. Each teacher was asked to identify one research-based practice that would expand his or her personal repertoire of instructional strategies and to introduce its use during the first three months of school. Follow-up discussions were held by content-area teachers and specialists, as well as by the specialists who met as a group to share ideas generated by the teachers with whom they worked.
- One respondent identified an important use for these materials: to validate the instructional practices that teachers already employ. In his words, 'it is as important for teachers to know what they know as well as what they still have to learn'.
- Teachers in one district reviewed and discussed the research findings, then received training and follow-up support in strategies in which they were interested.
- One principal, while expressing concern about the time that teachers in her school spent at the photocopying machine,

kept a copy of the *Handbook* by the machine. She reported that teachers liked the short format, which allowed them to read quickly about one of the practices.

- Another suggestion made by teachers was the use of the materials to help less-experienced teachers 'take the rough edges off'. More-experienced teachers would work collaboratively with them to help the newer teachers expand and refine their repertoire of strategies.

The context: a school culture for effective staff development

Experience has shown that teachers need time to absorb new information, observe and discuss new practices, and participate in the training needed to become confident with new techniques. This often means changes in traditional schedules to give teachers regular opportunities to team with their colleagues, both to acquire new skills and to provide instruction. As schools continue the task of improving student achievement by expanding the knowledge base of teachers, the need to restructure schools will become more and more apparent.

Successful use of the knowledge base on improving student learning in mathematics, as in the case of all the other subjects included in the *Handbook*, relies heavily on effective staff development. As Dennis Sparks, executive director of the National Staff Development Council, says in his *Handbook* chapter:

If teachers are to consistently apply in their classrooms the findings of the research described in this Handbook, high-quality staff development is essential. This professional development, however, must be considerably different from that offered in the past. It must not only affect the knowledge, attitudes, and practices of individual teachers, administrators, and other school employees, but it must also alter the cultures and structures of the organizations in which those individuals work.

Changes needed in the culture of staff development include an increased focus on both organization development and individual development; an inquiry approach to the study of the teaching/learning process; staff development efforts driven by clear, coherent strategic plans; a greater focus on student needs and learning outcomes; and inclusion of both generic and content-specific pedagogical skills.

The contents of this booklet and the *Handbook of research on improving student achievement* can provide the basis for well-designed staff development activities. If schools provide generous opportunities for teacher learning and collaboration, teachers can and will improve teaching and learning in ways that truly benefit all students. To achieve that end, professional development must be viewed as an essential and indispensable part of the school improvement process.

Additional resources

Resources available through the Educational Research Service

Handbook of research on improving student achievement, second edition (207 pages, plus appendix). Edited by Gordon Cawelti, this publication gives teachers, administrators and others access to the knowledge base on instructional practices that improve student learning in all the major subject areas from kindergarten to the end of secondary education, including mathematics. The *Handbook*, originally published in 1995, has been updated by the original authors, who are respected authorities in their content areas. Thorough reviews of the recent research have led to the addition of new practices and expanded insight into existing practices. An appendix covers research-based practices in beginning reading instruction.

- ***Improving student achievement in mathematics*** (28-page booklet). This booklet contains the entire mathematics chapter of the *Handbook of research on improving student achievement*, written by Douglas A. Grouws and Kristin J. Cebulla. It includes an introduction by Gordon Cawelti and a section on ideas for expanding teachers' ability to use research-based instructional practices.
- ***Improving student achievement in mathematics*** (two 30-minute videotapes). These videotapes illustrate each of the ten instructional practices described in the mathematics chapter of the *Handbook*, using classroom scenes and interviews with teachers and school administrators in the Cedar Rapids School District, Iowa, and the Alexandria City Schools, Virginia. The teachers' insights based on their actual experience using these research-based practices can serve as a springboard for powerful staff development activities that will spark discussion and further exploration. The ten practices are presented in self-contained segments, giving users the option of viewing one practice and then studying that practice in detail before exploring additional practices.

ERS Info-Files

Each *ERS Info-File* contains 70–100 pages of articles from professional journals, summaries of research studies and related literature concerning the topic, plus an annotated bibliography that includes an Educational Resources Information Center (ERIC-CIJE) search.

- *Math education and curriculum development*. Examines the implementation of the curriculum standards for mathematics, including models for integrating the standards and related impacts on students and teachers.
- *Math manipulatives and calculators*. Describes the use of concrete objects to teach mathematical concepts. Includes suggestions for materials, the scope of use of manipulatives, structuring manipulatives into lesson plans and use of computers. Discusses the rationale for using calculators to teach mathematical concepts.
- *Problem solving in math and science*. Reviews effective methods and strategies for teaching problem solving from kindergarten to grade 12. Materials include ideas for activities as well as grading methods.

Additional sources of information

Every child mathematically proficient: an action plan. This action paper was developed by the Learning First Alliance, an organization of twelve leading national education associations. It sets forth recommendations for curriculum changes, professional development initiatives, parent involvement efforts and research-based reforms. 24 pages. Price: $3.00. Order from National Education Association Professional Library order desk: (1-800) 229-4200.

Improving teaching and learning in science and mathematics. Illustrates how constructivist ideas can be used by science and mathematics educators for research and the further improvement of mathematics practice. 1996. Available from Teachers College Press, Teachers College, Columbia University, P.O. Box 20, Williston, VT 05495, USA.
Telephone: (1-802) 864-7626.

Mathematics, science, & technology education programs that work, and *Promising practices in mathematics and science*. Published by the United States Department of Education. The first volume describes programmes from the Department's National Diffusion Network; the second describes successful programmes identified by the Office of Educational Research and Improvement. Price: $21.00 for the two-volume set. Stock No. 065-000-00627-8. Available from Superintendent of Documents, P.O. Box 371954, Pittsburgh, PA 15250-7954, USA. Telephone: (1-202) 512-1800; fax: (1-202) 512-2250.

Curriculum and evaluation standards for school mathematics. Describes fifty-four standards developed by the National Council of Teachers of Mathematics to 'create a coherent vision of mathematical literacy and provide standards to guide the revision of the mathematics curriculum in the next decade'. 1989. 258 pages. $25.00. Available from National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1593, USA. Telephone: (1-703) 620-9840; fax: (1-703) 476-2970.

Eisenhower National Clearinghouse (Ohio State University). Part of a network funded by the United States Department of Education, which together with ten regional science and mathematics consortia, collaborates to identify and disseminate exemplary materials, to provide technical assistance about teaching methods and tools to schools, teachers and administrators, and to work with other organizations trying to improve mathematics and science education.
Online: www.enc.org

National Center for Improving Student Learning and Achievement in Mathematics and Science, Wisconsin Center for Educational Research, University of Wisconsin-Madison. Publications include the quarterly newsletter *Principled practice*, which examines educators' observations and concerns about issues in mathematics and science education. 1025 West Johnson Street, Madison, WI 53706, USA.
Telephone: (1-608) 265-6240; fax: (1-608) 263-3406;
e-mail: **ncisla@mail.soemadison.wisc.edu**;
web site: www.wcer.wisc.edu/ncisla

Related websites

- ERIC Clearinghouse for Science, Mathematics, and Environmental Education
  www.ericse.org/sciindex.html
- The Regional Alliance for Mathematics and Science Education
  http://ra.terc.edu/alliance/HubHome.html
- National Council of Teachers of Mathematics
  www.nctm.org

References

American Association of University Women. 1998. *Gender gaps: where schools still fail our children*. Washington, DC, AAUW.

Atanda, D. 1999. *Do gatekeeper courses expand education options?* Washington, DC, National Center for Education Statistics. (NCES 1999303.)

Aubrey, C. 1997. *Mathematics teaching in the early years: an investigation of teachers' subject knowledge*. London, Falmer Press.

Ball, D. 1993. With an eye on the mathematical horizon: dilemmas of teaching elementary school mathematics. *Elementary school journal* (Chicago, IL), vol. 93, p. 373–97.

Boaler, J. 1998. Open and closed mathematics: student experiences and understandings. *Journal for research in mathematics education* (Reston, VA), vol. 29, p. 41–62.

Brownell, W.A. 1945. When is arithmetic meaningful? *Journal of educational research* (Washington, DC), vol. 38, p. 481–98.

—. 1947. The place of meaning in the teaching of arithmetic. *Elementary school journal* (Chicago, IL), vol. 47, p. 256–65.

Carpenter, T.P., et al. 1988. Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for research in mathematics education* (Reston, VA), vol. 19, p. 385–401.

—. 1989. Using knowledge of children's mathematics thinking in classroom teaching: an experimental study. *American educational research journal* (Washington, DC), vol. 26, p. 499–531.

—. 1998. A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for research in mathematics education* (Reston, VA), vol. 29, p. 3–20.

Cobb, P.; Yackel, E.; Wood, T. 1992. A constructivist alternative to the representational view of mind in mathematics education. *Journal for research in mathematics education* (Reston, VA), vol. 23, p. 2–23.

Cobb, P., et al. 1991. Assessment of a problem-centered second-grade mathematics project. *Journal for research in mathematics education* (Reston, VA), vol. 22, p. 3–29.

—. 1992. Characteristics of classroom mathematics traditions: an interactional analysis. *American educational research journal* (Washington, DC), vol. 29, p. 573–604.

Cognition and Technology Group. 1997. *The Jasper Project: lessons in curriculum, instruction, assessment, and professional development.* Mahwah, NJ, Erlbaum.

Cohen, E.G. 1994. Restructuring the classroom: conditions for productive small groups. *Review of educational research* (Washington, DC), vol. 64, p. 1–35.

Davidson, N. 1985. Small group cooperative learning in mathematics: a selective view of the research. *In:* Slavin, R., ed. *Learning to cooperate, cooperating to learn*, p. 211–30. New York, Plenum Press.

Davis, M. 1990. *Calculating women: precalculus in context*. Paper presented at the Third Annual Conference on Technology in Collegiate Mathematics, Columbus, OH, November.

Drijvers, P.; Doorman, M. 1996. The graphics calculator in mathematics education. *Journal of mathematical behavior* (Stamford, CT), vol. 15, p. 425–40.

Dunham, P.H.; Dick, T.P. 1994. Research on graphing calculators. *Mathematics teacher* (Reston, VA), vol. 87, p. 440–45.

Fawcett, H.P. 1938. *The nature of proof: a description and evaluation of certain procedures used in senior high school to develop an understanding of the nature of proof.* 1938 Yearbook of the National Council of Teachers of Mathematics. New York, Columbia University, Teachers College.

Fennema, E.; Carpenter, T.P.; Peterson; P.L. 1989. Learning mathematics with understanding: cognitively guided instruction. In: Brophy, J., ed. *Advances in research on teaching*, p. 195–221. Greenwich, CT, JAI Press.

Fennema, E., et al. 1993. Using children's mathematical knowledge in instruction. *American educational research journal* (Washington, DC), vol. 30, p. 555–83.

—. 1996. A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for research in mathematics education* (Reston, VA), vol. 27, p. 403–34.

Flanders, J.R. 1987. How much of the content in mathematics textbooks is new?' *Arithmetic teacher* (Reston, VA), vol. 35, p. 18–23.

Flores, A.; McLeod, D.B. 1990. *Calculus for middle school teachers using computers and graphing calculators*. Paper presented at the Third Annual Conference on Technology in Collegiate Mathematics, Columbus, OH, November.

Fuson, K.C. 1992. Research on whole number addition and subtraction. *In:* Grouws, D.A., ed. *Handbook of research on mathematics teaching and learning*, p. 243–75. New York, Macmillan.

Fuson, K.C.; Briars, D.J. 1990. Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for research in mathematics education* (Reston, VA), vol. 21, p. 180–206.

Giamati, C.M. 1991. *The effect of graphing calculator use on students' understanding of variations of their graphs.* Doctoral dissertation, University of Michigan. *Dissertation Abstracts International*, vol. 52, 103A. (University Microfilms No. AAC 9116100.)

Good, T.L.; Grouws, D.A.; Ebmeier, H. 1983. *Active mathematics teaching.* New York, Longman.

Greeno, J.G. 1991. Number sense as situated knowing in a conceptual domain. *Journal for research in mathematics education* (Reston, VA), vol. 22, p. 170–218.

Grouws, D.A.; Smith, M.S. In press. Findings from NAEP on the preparation and practices of mathematics teachers. *In:* Silver, E.A.; Kenney, P., eds. *Results from the Seventh Mathematics Assessment of the National Assessment of Educational Progress.* Reston, VA, National Council of Teachers of Mathematics.

Groves, S.; Stacey, K. 1998. Calculators in primary mathematics: exploring number before teaching algorithms. *In:* Morrow, L.J., ed. *The teaching and learning of algorithms in school mathematics*, p. 120–29. Reston, VA, National Council of Teachers of Mathematics.

Harvey, J.G. 1993. *Effectiveness of graphing technology in a pre-calculus course: the 1988–89 field test of the C3PC materials.* Paper presented at the Technology in Mathematics Teaching Conference, Birmingham, UK, September.

Hawkins, E.F.; Stancavage, F.B.; Dossey, J.A. 1998. *School policies and practices affecting instruction in mathematics: findings from the National Assessment of Educational Progress.* Washington, DC, National Center for Educational Statistics. (NCES 98-495.)

Heid, M.K. 1988. Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for research in mathematics education* (Reston, VA), vol. 19, p. 3–25.

Hembree, R.; Dessart, D.J. 1986. Effects of hand-held calculators in pre-college mathematics education: a meta-analysis. *Journal for research in mathematics education* (Reston, VA), vol. 17, p. 83–99.

—. 1992. Research on calculators in mathematics education. *In:* Fey, J.T., ed. *Calculators in mathematics education*. 1992 Yearbook of the National Council of Teachers of Mathematics, p. 22–31. Reston, VA, National Council of Teachers of Mathematics.

Hiebert, J.; Carpenter, T. 1992. Learning and teaching with understanding. In: Grouws, D.A., ed. *Handbook of research on mathematics teaching and learning*, p. 65–97. New York, Macmillan.

Hiebert, J.; Wearne, D. 1992. Links between teaching and learning place value with understanding in first grade. *Journal for research in mathematics education* (Reston, VA), vol. 22, p. 98–122.

—. 1993. Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American educational research journal* (Washington, DC), vol. 30, p. 393–425.

—. 1996. Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and instruction* (Hillsdale, NJ), vol. 14, p. 251–83.

Hiebert, J., et al. 1997. *Making sense: teaching and learning mathematics with understanding.* Portsmouth, NH, Heinemann.

Husén, T. 1967. *International study of achievement in mathematics*, vol. 2. New York, Wiley.

Kamii, C. 1985. *Young children reinvent arithmetic: implications of Piaget's theory*. New York, Teachers College Press.

—. 1989. *Young children continue to reinvent arithmetic: implications of Piaget's theory*. New York, Teachers College Press.

—. 1994. *Young children continue to reinvent arithmetic in 3rd grade: implications of Piaget's theory*. New York, Teachers College Press.

Keeves, J.P. 1976. Curriculum factors influencing school learning. *Studies in educational evaluation* (Kidlington, UK), vol. 2, p. 167–84.

—. 1994. *The world of school learning: selected key findings from 35 years of IEA research*. The Hague, Netherlands, International Association for the Evaluation of Educational Achievement (IEA).

Kilpatrick, J. 1992. A history of research in mathematics education. In: Grouws, D.A., ed. *Handbook of research on mathematics teaching and learning*, p. 3–38. New York, Macmillan.

Knapp, M.S.; Shields, P.M.; Turnbull, B.J. 1995. Academic challenge in high-poverty classrooms. *Phi Delta Kappan* (Bloomington, IN), vol. 77, p. 770–76.

Koehler, M.; Grouws, D.A. 1992. Mathematics teaching practices and their effects. *In:* Grouws, D.A., ed. *Handbook of research on mathematics teaching and learning,* p. 115–26. New York, Macmillan.

Kulm, G.; Morris, K.; Grier, L. 1999. *Middle grade mathematics textbooks: a benchmarks-based evaluation*. Washington, DC, American Association for the Advancement of Science.

Labinowicz, E. 1985. *Learning from students: new beginnings for teaching numerical thinking*. Menlo Park, CA, Addison-Wesley.

Laborde, C. 1994. Working in small groups: a learning situation? *In:* Biehler, R., et al., eds. *Didactics of mathematics as a scientific discipline*, p. 147–58. Dordrecht, Netherlands, Kluwer Academic Publishers.

Leinenbach, M.; Raymond, A.M. 1996. A two-year collaborative action research study on the effects of a 'hands-on' approach to learning algebra. *In:* Jakubowski, E., ed. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Panama City, FL. (ERIC Document Reproduction; Service No. ED 400 178.)

Mack, N.K. 1990. Learning fractions with understanding: building on informal knowledge. *Journal for research in mathematics education* (Reston, VA), vol. 21, p. 16–32.

Markovits, Z.; Sowder, J. 1994. Developing number sense: an intervention study in grade 7. *Journal for research in mathematics education* (Reston, VA), vol. 25, p. 4–29.

McKnight, C.C., et al. 1987. *The underachieving curriculum*. Champaign, IL, Stipes.

Mullis, I.V.S.; Jenkins, F.; Johnson, E.G. 1994. *Effective schools in mathematics: perspectives from the NAEP 1992 assessment.* Washington, DC, United States Department of Education, Office of Educational Research and Improvement. (Report No. 23-RR-01.)

National Center for Education Statistics. 1996. *Pursuing excellence: a study of U.S. eighth-grade mathematics and science teaching, learning, curriculum, and achievement in international context.* Washington, DC, United States Department of Education. (NCES Report 97-198.)

—. 1997. *Pursuing excellence: a study of U.S. fourth-grade mathematics and science achievement in international context.* Washington, DC, United States Department of Education. (NCES Report 97-255.)

—. 1998. *Pursuing excellence: a study of U.S. twelfth-grade mathematics and science achievement in international* *context*. Washington, DC, United States Department of Education. (NCES Report 98-049.)

National Council of Teachers of Mathematics. 1989. *Curriculum and valuation standards for school mathematics*. Reston, VA, NCTM.

Penglase, M.; Arnold, S. 1996. The graphics calculator in mathematics education: a critical review of recent research. *Mathematics education research journal* (Campbelltown, Australia), vol. 8, p. 58–90.

Resnick, L.B. 1980. The role of invention in the development of mathematical competence. *In:* Kluwe, R.H.; Spada, H., eds. *Developmental models of thinking*, p. 213–44. New York, Academic Press.

Resnick, L.B.; Omanson, S.F. 1987. Learning to understand arithmetic. *In:* Glaser, R., ed. *Advance in instructional psychology*, vol. 3, p. 41–95. Hillsdale, NJ, Lawrence Erlbaum Associates.

Reys, B.J.; Barger, R.H. 1994. Mental computation: issues from the United States perspective. *In:* Reys, R.E.; Nohda, N., eds. *Computational alternatives for the twenty-first century*, p. 31–47. Reston, VA, National Council of Teachers of Mathematics.

Reys, B.J., et al. 1991. *Developing number sense in the middle grades*. Reston, VA, National Council of Teachers of Mathematics.

Rich, B.S. 1991. *The effects of the use of graphing calculators on the learning of function concepts in precalculus mathematics*. Doctoral dissertation, University of Iowa. *Dissertation Abstracts International*, vol. 52, 835A. (University Microfilms No. AAC 9112475.)

Ruthven, K. 1990. The influence of graphic calculator use on translation from graphic to symbolic forms. *Educational studies in mathematics* (Dordrecht, Netherlands), vol. 21, p. 431–50.

Schmidt, W.H.; McKnight, C.C.; Raizen, S.A. 1997. *A splintered vision: an investigation of U.S. science and mathematics education*. Dordrecht, Netherlands, Kluwer Academic Publishers.

Secada, W.G. 1992. Race, ethnicity, social class, language, and achievement in mathematics. *In:* Grouws, D.A., ed. *Handbook of research on mathematics teaching and learning*, p. 623–60. New York, Macmillan.

Skemp, R.R. 1978. Relational understanding and instrumental understanding. *Arithmetic teacher* (Reston, VA), vol. 26, p. 9–15.

Slavin, R.E. 1990. Student team learning in mathematics. *In:* Davidson, N., ed. *Cooperative learning in math: a handbook for teachers,* p. 69–102. Reading, MA, Addison-Wesley.

—. 1995. *Cooperative learning: theory, research, and practice.* 2nd edition. Boston, Allyn & Bacon.

Slavit, D. 1996. Graphing calculators in a 'hybrid' algebra II classroom. *For the learning of mathematics: an international journal of mathematics education* (Montreal, Canada) vol. 16, p. 9–14.

Smith, B.A. 1996. *A meta-analysis of outcomes from the use of calculators in mathematics education.* Doctoral dissertation, Texas A&M University at Commerce. *Dissertation Abstracts International,* vol. 58, 03.

Sowder, J. 1992*a*. Estimation and number sense. *In:* Grouws, D.A., ed. *Handbook of research on mathematics teaching and learning,* p. 371–89. New York, Macmillan.

—. 1992*b*. Making sense of numbers in school mathematics. *In:* Leinhardt, R.; Putnam, R.; Hattrup, R., eds. *Analysis of arithmetic for mathematics education,* p. 1–51. Hillsdale, NJ, Lawrence Erlbaum Associates.

Sowell, E.J. 1989. Effects of manipulative materials in mathematics instruction. *Journal for research in mathematics education* (Reston, VA), vol. 20, p. 498–505.

Stacey, K.; Groves, S. 1994. *Calculators in primary mathematics.* Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Indianapolis, IN, April.

Stigler, J.W.; Hiebert, J. 1997. Understanding and improving classroom mathematics instruction. *Phi Delta Kappan* (Bloomington, IN), vol. 79, p. 14–21.

Stigler, J.W., et al. 1999. *The TIMSS videotape study: methods and findings from an exploratory research project on eighth grade mathematics instruction in Germany, Japan, and the United States.* Washington, DC, National Center for Education Statistics. (NCES 99-130.)

Suarez, T.M., et al. 1991. Enhancing effective instructional time: a review of research. *Policy brief*, vol. 1, no. 2. Chapel Hill, NC, North Carolina Educational Policy Research Center.

Suydam, M.N.; Higgins, J.L. 1977. *Activity-based learning in elementary school mathematics: recommendations from research.* Columbus, OH, ERIC Center for Science, Mathematics, and Environmental Education.

Thompson, P.W. 1992. Notations, conventions, and constraints: contributions of effective uses of concrete materials in elementary mathematics. *Journal for research in mathematics education* (Reston, VA), vol. 23, p. 123–47.

Van Engen, H. 1949. An analysis of meaning in arithmetic. *Elementary school journal* (Chicago, IL), vol. 48, p. 395–400.

Varelas, M.; Becker, J. 1997. Children's developing understanding of place value: semiotic aspects. *Cognition and instruction* (Hillsdale, NJ), vol. 15, p. 265–86.

Wearne, D.; Hiebert, J. 1988. A cognitive approach to meaningful mathematics instruction: testing a local theory using decimal numbers. *Journal for research in mathematics education* (Reston, VA), vol. 19, p. 371–84.

Webb, N.M. 1991. Task-related verbal interaction and mathematics learning in small groups. *Journal for research in mathematics education* (Reston, VA), vol. 22, p. 366–89.

Webb, N.M.; Troper, J.D.; Fall, R. 1995. Constructive activity and learning in collaborative small groups. *Journal of educational psychology* (Washington, DC), vol. 87, p. 406–423.

Wilson, M.R.; Krapfl, C.M. 1994. The impact of graphics calculators on students' understanding of function. *Journal of computers in mathematics and science teaching* (Charlottesville, VA), vol. 13, p. 252–64.

Wood, T. 1999. Creating a context for argument in mathematics class. *Journal for research in mathematics education* (Reston, VA), vol. 30, p. 171–91.

Wood, T.; Cobb, P.; Yackel, E. 1995. Reflections on learning and teaching mathematics in elementary school. *In:* Steffe, L.P.; Gale, J., eds. *Constructivism in education*, p. 401–22. Hillsdale, NJ, Lawrence Erlbaum Associates.

Wood, T.; Sellers, P. 1996. Assessment of a problem-centered mathematics program: 3rd grade. *Journal for research in mathematics education* (Reston, VA), vol. 27, p. 337–53.

—. 1997. Deepening the analysis: longitudinal assessment of a problem-centered mathematics program. *Journal for research in mathematics education* (Reston, VA), vol. 28, p. 163–86.

Wood, T., et al. 1993. Rethinking elementary school mathematics: insights and issues. *Journal for research in mathematics education* (Reston, VA), monographs, 6.

Yackel, E.; Cobb, P.; Wood, T. 1991. Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for research in mathematics education* (Reston, VA), vol. 22, p. 390–408.

NOTES

EDUCATIONAL PRACTICES SERIES–4

The International Bureau of Education—IBE

An international centre for the content of education, the IBE was founded in Geneva in 1925 as a private institution. In 1929, it became the first intergovernmental organization in the field of education. In 1969, the IBE joined UNESCO as an integral, yet autonomous, institution with three main lines of action: organizing the sessions of the International Conference on Education; collecting, analysing and disseminating educational documentation and information, in particular on innovations concerning curricula and teaching methods; and undertaking surveys and studies in the field of comparative education.

At the present time, the IBE: (a) manages *World data on education*, a databank presenting on a comparative basis the profiles of national education systems; (b) organizes courses on curriculum development in developing countries; (c) collects and disseminates through its databank INNODATA notable innovations on education; (d) co-ordinates preparation of national reports on the development of education; (e) administers the Comenius Medal awarded to outstanding teachers and educational researchers; and (f) publishes a quarterly review of education—*Prospects*, a quarterly newsletter—*Educational innovation and information*, a guide for foreign students—*Study abroad*, as well as other publications.

In the context of its training courses on curriculum development, the Bureau is establishing regional and sub-regional networks on the management of curriculum change and developing a new information service—a platform for the exchange of information on content.

The IBE is governed by a Council composed of representatives of twenty-eight Member States elected by the General Conference of UNESCO.

http://www.ibe.unesco.org