

RECOMMENDATION No. 43
TO THE MINISTRIES OF EDUCATION
concerning
THE TEACHING OF MATHEMATICS IN SECONDARY SCHOOLS
(1956)

The International Conference on Public Education,

Convened in Geneva by the United Nations Educational, Scientific and Cultural Organization, and the International Bureau of Education, and having assembled on the ninth of July, nineteen hundred and fifty-six for its nineteenth session, adopts on the seventeenth of July, nineteen hundred and fifty-six the following Recommendation:

The Conference,

Considering that mathematics has always had undisputed cultural and practical value and an important role in scientific, technical and economic progress, and that the present situation in the mathematical field is without precedent,

Considering that mathematical training is a benefit and a privilege for every human being, whatever his race, sex, condition, and activities,

Considering that to ensure world progress and prosperity the general mathematical level must be raised in line with higher technical and scientific expansion,

Considering that the various civilizations have all played a part in the creation and development of mathematics,

Considering that psychology recognises that practically every human being is capable of a certain amount of mathematical activity, and that there is no reason to believe that girls are less able to study mathematics than boys,

Considering that the teaching of mathematics is becoming increasingly scientific and efficient,

Considering that Recommendation No. 31 concerning the Introduction to Mathematics in Primary Schools, adopted by the XIIIth International Conference on Public Education, needs to be extended,

Submits to the Ministries of Education of the several countries the following recommendation:

Aims of mathematics teaching

1) Throughout both general and vocational secondary education every possible endeavour should be made to achieve the aims of mathematics teaching bearing on intellectual capacity and the formation of character. These aims lead to the logical process in action (thinking, analysing, abstracting, schematizing, reasoning deductively, generalizing, specializing, applying, criticizing, etc.), to the qualities of rational thought and its expression (order, precision, clarity, concision, etc.), to observation, the concepts of space and number, intuition and imagination in the abstract field, attention and the power to concentrate, perseverance and the habit of ordered effort, and finally to the formation of the scientific outlook (objectivity, intellectual honesty, interest in research, etc.).

2) The operations of a practical order, the adaptation to the natural environment, and the need to understand the problems raised by technical, economic and social activities, make it more and more necessary to understand current mathematical processes (computation, fundamental geometry, geometrical representation, equations, formulas, functions, tables and graphs). These fundamental ideas and means also play a part in an increasing number of professions.

3) Mathematics and the appropriate process of thought should be regarded as an essential part of the education of a modern person, even if his work is not of a scientific or technical nature. The teaching of mathematics, closely coordinated with that of other subjects, should lead pupils to understand the part played by mathematics in the scientific and philosophical conceptions of the modern world.

4) One of the main aims of the advanced course of mathematics in upper secondary classes should be preparation for scientific or technical studies at higher level, whose mathematical basis is steadily increasing.

R 43

Place given to mathematics

5) Mathematics, a required subject in the various classes of the lower stage of secondary education, should have an adequate number of hours allotted to it.

6) In the upper stage of the science sections, ample time should be allotted to the mathematics course.

7) It is desirable that pupils with special aptitudes for the study of science should have the opportunity of following a more advanced syllabus and of working by themselves.

8) Instruction in mathematics of a cultural rather than a purely technical kind should be offered, at least as an option, in countries where in certain sections (arts sections, for example) mathematics is not a required subject.

9) In any marking scheme the weight given to mathematics should be proportional to the importance of the subject. Where mathematics is a required subject, and particularly in science sections, it should be considered as one of the main branches, especially for class promotion and the award of leaving certificates.

Syllabuses

10) The mathematics syllabus of any particular section of a secondary school conform to the general aims of mathematics teaching and the particular needs of the section.

11) Mathematics syllabuses should be kept up-to-date and adapted to the progress of science and the needs of technical activities and modern life, by discarding outdated questions. Special consideration should be given to the fact that some countries, in order to raise the level of upper secondary syllabuses, have introduced analytical geometry, infinitesimal calculus, statistics and probability, and attach increasing importance to the study of functions and vectors as well as to the applications of mathematics.

12) The difficulty and the extent of the subject matter to be taught should be related to the mental age corresponding to each class, and to the pupils' interests and needs. If it is desirable to teach more advanced work to gifted pupils, care should be taken not to discourage the less gifted pupils by imposing on them subject matter too complex for their intellectual ability.

13) It is desirable to draw up curricula so as to base the teaching of mathematics on functional topics which bring out the general notions while coordinating the separate branches.

14) In this connection, it is also desirable to determine, by careful experiment, to what extent the multiform structure of modern mathematics can be made to enrich secondary education.

15) It is desirable that teachers should be given freedom to extend the scope of basic syllabuses by optional additions.

Methods

16) Where instructions on methods are given, they should take the form of advice and suggestions to bring teaching into line with advances in the psychology of intelligence and the teaching of mathematics, and with the nature and use of mathematics, a theoretical science with origins in the real world and effective power for action upon it.

17) Every effort should be made to stimulate and encourage pupils to learn mathematics by active participation in the development of the subject.

18) It is essential:

a) to awaken and maintain the pupils' interest in both mathematics and its applications;

b) to pay attention to the pupils' own way of thinking in mathematics;

c) to adapt teaching to the pupils' individual capacities and intellectual growth, and gradually to adapt it to the needs of their future activities.

19) It is essential:

a) to proceed as often as possible from the concrete to the abstract, especially in the lower classes, and whenever possible to make use of actual, figurative or imagined experimentation, in order to suggest definition or demonstration;

b) to take into account that mathematical knowledge arises through the interiorisation of concrete activity and the development of operations;

c) to make use of questions which have a practical origin, not only in order to emphasise the utility of mathematics, but also to motivate theoretical developments.

R 43

20) It is important:

a) to lead the pupils to formulate ideas and discover mathematical relations and properties for themselves, rather than impose on them ready-made adult thinking;

b) to ensure the assimilation of ideas and operational processes before introducing formal generalisation;

c) to convert into skills only those processes which have been fully assimilated.

21) It is essential:

a) to give pupils experience of mathematical entities and relations first, and only then to introduce them to deductive reasoning;

b) to extend deductive processes in mathematics gradually;

c) to train pupils to make up problems, to search for and utilize data, and to evaluate results;

d) to develop the subject heuristically rather than to teach it dogmatically;

e) to make pupils conscious of the structure of a hypothetical deductive theory in which, on the basis of postulates, the theorems are built up by demonstrations and new terms are introduced by definitions, so as to lead up to deductive logical treatment.

22) It is necessary:

a) to study the mistakes pupils make, and to see in them a means of understanding their thought processes in mathematics;

b) to lead pupils to control and to correct their work themselves;

c) to give pupils a sense of approximation, order of magnitude, and the reasonableness of results;

d) to give preference to thought and reasoning rather than to mechanical drill and learning by heart, and to limit the use of memory to fixing fundamental results; *e)* to set examination papers which will call for mathematical skill rather than mere cramming.

23) It is important:

a) to encourage individual modes of expression, even if only approximate, and to improve them gradually;

b) to lead pupils towards precision and exactitude through their need to communicate effectively with other persons and to clarify their thoughts;

c) to foster individual research and initiative as much as team work;

d) to increase the number of pupils interested in mathematics and to assist in developing their formation and knowledge, by organizing clubs, lectures, competitions and other activities of an optional nature, and by providing them with the necessary books and periodicals.

24) It is essential:

a) to emphasise the intrinsic unity of mathematics, to cease putting its various branches into watertight compartments, and to bring together the various ways of solving a problem;

b) to indicate the main stages in the historical growth of the mathematical ideas and theories being studied.

25) It is essential:

a) to maintain the coordination of mathematics with the sciences which make use of mathematics;

b) to utilize the requirements of mathematical thought in order to increase accuracy, clarity and concision of language;

c) to keep the link intact between mathematics, on the one hand, and life and reality, on the other.

Teaching materials

26) The evolution of the methodology of mathematics calls for the adaptation of textbooks. Besides textbooks giving a graded introduction to abstract ideas, the pupil should have access to books for revision where the subject matter is taken up again and organized on a higher level. Books of reference, supplementary material, popular handbooks, periodicals, etc., should be available in every class library. Such documentation should be adapted to the needs of the different sections and pay due heed to practical and technical needs, theoretical developments, and educational considerations.

27) Audio-visual aids, concrete mathematical models (from daily life, constructed by pupils or teachers, or manufactured by commercial firms), play an increasingly large part in teaching, and advantage should be taken of their use to enable pupils to acquire mathematical abstractions actively.

Teachers

28) In mathematics, perhaps even more than in other subjects, the role of the teacher is of prime importance. The recruitment, training and further training of mathematics teachers should be the object of special care on the part of the education authorities.

29) Teachers entrusted with the teaching of mathematics in secondary schools should have studied mathematics at a level considerably beyond that to which they will be required to teach. These studies should include not only theoretical mathematics but also some applied mathematics, an outline of the history of mathematical thought, the methodology of mathematical science itself, and the study of elementary mathematics considered from a higher point of view.

30) Care should be taken to ensure that all pupils in the earlier years and the less-able pupils in the later years are taught by the best possible teachers.

31) Adequate professional and psychological training should be regarded as a necessary complement of the teacher's mathematical studies, and should be based on clear and mature understanding of the principles and objectives of humanistic education. It should give emphasis to the structural evolution of intelligence in relation to the development of mathematical thought. It should include the relations of the concrete and the abstract and give proper place to the methodology of models in mathematics teaching. The prospective teacher should be trained to observe and experiment in the teaching of mathematics. Above all, he should be interested in adolescents and their aspirations and thus enabled to act as their leader and guide.

32) Serving mathematics teachers should be in a position to keep abreast of modern developments in the theoretical mathematical sciences important present-day applications of mathematics, and recent advances in the teaching of the subject. Steps should be taken to facilitate teachers' further training (lectures, vacation courses, seminars, workshops, practical courses, publications, etc.).

33) Suggestions from specialist inspectors and educational advisers and the example of the work of experienced teachers, are excellent ways of improving the teaching.

34) In modern society secondary mathematics teachers should enjoy the esteem and standing to which their scientific studies and their vocation entitle them.

35) The adequate teaching of mathematics being an essential part of education in all countries, the teaching profession must attract sufficient qualified teachers in this subject. This is of special importance since an adequate supply of mathematics teachers governs the scientific, technical, economic and social development of all nations.

International collaboration

36) Governments and international cultural and educational bodies such as Unesco, the International Bureau of Education, the International Commission on the Teaching of Mathematics, and the International Commission for the Study and Improvement of the Teaching of Mathematics, should promote by all means at their disposal (publications, lectures, meetings, exhibitions, travel and study abroad, etc.) the international exchange of ideas, work, research and results obtained in the teaching of mathematics, so that the youth of the whole world may benefit without delay from the experience and progress of teachers in all countries.