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VIEWPOINTS/CONTROVERSIES

OPEN FILE: PIAGET AND EDUCATION

TRENDS/CASES

PROFILES OF FAMOUS EDUCATORS

Please address all editorial correspondence to: Juan Carlos Tedesco, Editor, Prospects, International Bureau of Education, P.O. Box 199, 1211 Geneva 20, Switzerland. Email: JC.TEDESCO@unesco.org

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EDITORIAL

In 1996 we will celebrate the 250th anniversary of the birth of Johann H. Pestalozzi and the 100th anniversary of the births of Jean Piaget, Célestin Freinet and Lev Vygotsky.[1] A large number of activities have been foreseen to commemorate the achievements and thoughts of these four important educators, many of them sponsored institutionally by UNESCO's International Bureau of Education. Beyond the actual significance of their achievements, mentioning simultaneously the thoughts of Piaget, Freinet, Vygotsky and Pestalozzi confronts us with an analysis of one of the most significant problems of modern education: the link between scientific research—basically represented by the presence of Jean Piaget and Lev Vygotsky—and educational innovation—as represented by Pestalozzi and Freinet.

The name of Jean Piaget is closely linked with that of the International Bureau of Education and UNESCO. In company with his friend and colleague, the Catalan Pedro Roselló, Piaget was the director of the IBE from 1929 until 1968, a period during which relationships between peoples underwent a profound change that modified international co-operation into a necessary but, at the same time, a complicated and difficult task. Even though Piaget's public activity played a secondary role when compared to his scientific output, it is not without importance to state—at a time when we are commemorating the 100th anniversary of his birth—that Piaget maintained throughout his life a strong commitment to international educational co-operation.

International co-operation and scientific research were, for Piaget, inseparable. His speeches and reports as Director of the IBE are a witness to this. Both the analysis and the proposals about education and teaching put forward by Piaget to direct educational activities in the countries belonging to the IBE were based on his hypotheses about cognitive development, arising from the outcomes of his scientific research. Even though the effect of Piaget's research on the psychology of learning has been very significant—and this edition of *Prospects* is evidence of it—we could not say the same about the impact on the educational practices which are used in educational activities. At this level, the present situ-

ation is very similar to the one that existed when Piaget published his theoretical findings: promoting a culture of peace in a world where violence is being used again to resolve conflicts, and developing high-quality education in a universal manner so that the production of knowledge is not concentrated in an anti-democratic way.

To confront these challenges—today, as in the past—means renewing the scientific foundations of teaching methods, content and teacher training. However, experience has shown us that an adequate understanding of cognitive development is a necessary, but insufficient, condition for educational renewal. The process of educational reform incorporates social, cultural and political aspects that must be incorporated in the research plan for cognitive development, particularly when the school system plays a significant role in this development. Here, the contributions of Pestalozzi and Freinet, bearing in mind the obvious and substantial differences of epoch and cultural context, represent a rich source of learning: in the first place, in both cases, we come across a strong political involvement with the democratization of education, from the point of view of the sectors towards which educational innovations were directed as well as from the point of view of the ultimate objectives of educational actions; in the second place, both of them recognized that the key factor in the learning process is pupil activity; and, in the third place, both of them had a similar concern for the overall character of educational activity where the cognitive, affective and the manual play equally important roles in the learning process, conceived as part of the overall process of training the personality.

Our experience of educational renewal has also shown us the need to place innovation in a key position in reform strategies. The challenge, however, consists of incorporating innovatory activities in partnership with educational institutions and not in any limited way. Introducing the individualization of the learning process in education systems that are at grips with vast numbers of the population is today more than ever one of the main educational problems that must be overcome. In this case, it is indispensable that specialists in cognitive development, politicians, administrators and educators work together. The complexity of the problem, which can be illustrated by the historical difficulty of overcoming it, tells us that there are no simple or standard solutions. Piaget, Freinet, Vygotsky and Pestalozzi have left us a heritage in which, beyond knowledge, we find an important legacy in terms of involvement with the solving of educational problems.

This edition of *Prospects* also contains a very interesting article by Jacques Lesourne on one of the burning issues in discussions about education and society: the problem of employment. The linkage between education and the world of work represents a problem which should be re-examined in relation to past experience and the new challenges presented by changes in production processes. *Prospects* also publishes an analysis of the phenomenon of school repetition in Lebanon and, in its section of profiles on famous educators, a study on one of the

most influential philosophers in European thought in the nineteenth century: Auguste Comte.

JUAN CARLOS TEDESCO

Note

1. On this subject, see: Michel Soëtard, 'Johann Heinrich Pestalozzi (1746–1827)' in *Prospects: thinkers on education* (Paris, UNESCO), vol. 3, nos. 1/2, 1994, p. 297–310; Alberto Munari, 'Jean Piaget (1896–1980)', ibid., p. 311–27; Louis Legrand, 'Célestin Freinet (1896–1966)' in *Prospects: thinkers on education,* vol. 1, nos. 1/2, 1993, p. 403–18; and Ivan Ivić, 'Lev S. Vygotsky (1896–1934)' in *Prospects: thinkers on education,* vol. 4, nos. 3/4, 1994, p. 761–85.

VIEWPOINTS/CONTROVERSIES

EDUCATION AND EMPLOYMENT

Jacques Lesourne

Writing for *Prospects* always poses a dilemma. Should one address the problem on a worldwide scale, at the risk of putting forward hasty generalizations and superficial distinctions, or would it be better to confine oneself to one geographical area or even one country, and run the risk of lapsing into the particular?

After giving the matter some consideration, I have deliberately opted for the second standpoint by confining myself to France. Even so, I shall address relations between education and employment in this country in such a way that my approach may help give food for thought to readers from other countries and continents.

From 1945 to 1967, the French economy was in a situation of full employment. The worlds of education and economics ignored each other and even had a certain aversion for one another: teachers suspected employers of exploiting their wage-earners, while employers were afraid that schools were only educating rebels against authority. Each one defended its territory and educators made sure that education and training were kept under their control. Their main concern was to democratize education by extending the period of study and raising the

Jacques Lesourne (France)
Since 1994, Jacques Lesourne has been Professor of Economics and Statistics at the Conservatoire national des arts et métiers and is currently President of Futuribles International. In the course of his career, he has been Director-General, then President of SEMA (1958-75), the European advisory group on information technology and business management, Director of the Interfuturs project at OECD (1976-79) and Managing Editor of the newspaper *Le Monde* (1991-94). In addition to being a Fellow of the Econometrics Society, he was, *inter alia*, Chairman of the Association française de sciences économiques and the International Federation of Operational Research Societies (1986-89). His many works include *Le calcul économique, théorie et applications* [Economic calculating: theory and practice] (1972), *Les mille sentiers de l'avenir* [The thousand roads to the future] (1981), *L'économie de l'ordre et du désordre* [The tidy and the untidy economy] (1991).

leaving level of the largest possible number. Employers in their turn accorded priority to growth and investment and, under pressure from the unions, to the codification of social practices. It was also a period when real wages and social welfare underwent a considerable improvement.

In 1967, unemployment made its appearance and reached 1%. It thereafter developed gradually until 1974, in spite of economic growth that remained strong, then flared up intermittently from 1974 to 1986, when it affected 10.5% of the working population. The more sustained growth at the end of the 1980s resulted in a decline of about 300,000 people in the number of unemployed, but the situation again became gloomier at the beginning of the present decade, the low point being reached in 1993, with 12.6% of the labour force out of work. A slight improvement has been recorded in the last two years.

An analysis of this period spanning almost thirty years (1967-1995) prompts three complementary questions to be asked from the standpoint of relations between education and employment:

1. How did French society, through its various operators, experience this rise in unemployment and what explanations did it mainly arrive at as the phenomenon developed?
2. What *a contrario* is the diagnosis that we can make of employment trends in France and the reasons why French elites have proved unable to agree on the causes of unemployment and have only proposed policies that have been indecisive, confused and contradictory and have tended to vacillate?
3. What, in these circumstances, and on the strength of French experience, appears to be the desirable lines of approach to an educational policy taking employment into account?

I hope that an analysis addressing these three questions in turn will help all my readers to gain a better understanding of the problems that are a source of concern to them, even if the political, economic and social reference context is profoundly different.

A brief history of the rise in unemployment

When, in 1969, Jacques Chaban-Delmas, who had been appointed Prime Minister by Georges Pompidou immediately after the presidential elections, had Parliament enact a law on the training of workers at the prompting of Jacques Delors, its purpose was not to combat unemployment but to improve the situation of wage-earners and increase their promotion prospects.

Five years later, the worsening of unemployment shortly after the first oil shortage was not ascribed to the qualification of workers or the cost of labour, but to the abrupt slowdown in growth caused by the rise in energy prices.

It was only in the latter half of the 1970s that a relationship started to be established between training and employment. Some people pointed out that alongside unemployment due to slow growth and unemployment due to adjustment resulting from the restructuring of the production system, 'classical'

unemployment was growing, owing to the fact that employers stopped recruiting because the additional costs, both immediate and anticipated, of such recruitment seemed to them higher than the expected earnings.[1] It was that situation which seems to explain the emergence of long-term unemployment among the unskilled, and especially among young people.

Since then, the existence of a link between training and employment opportunities has been recognized, and this is reflected in both co-operation and conflict between teachers and employers. There is conflict because the latter accuse schools of sending them unusable applicants (the implication being 'at that price'), whereas the former, while being mainly concerned for the future of their pupils, point out that the unemployment rate falls when the level of the diploma obtained rises. There is co-operation because training techniques are changing and practical operations between employers and technical teaching establishments are increasing at the shop-floor level.

The latter half of the 1970s also saw the emergence of the theme of the reduction in working hours. The basis for the reasoning was as follows: the amount of labour needed is governed by the level of GDP (gross domestic product). However, as a result of the size of the active population, the volume of labour supplied by individuals is higher. It is therefore necessary to reduce working hours in proportion to the ratio of labour demand to labour supply. But two questions are hardly ever raised. Do all workers provide an equivalent service? And what happens to workers' earnings when there is a reduction in working hours?

It is also suggested on the extreme right that women and immigrants should be sent back to their homes, even if this means a reduction in the labour supply.

In 1981, after the election of François Mitterrand to the French presidency and the coming to power of a new political majority, unemployment was chiefly ascribed to technological progress, which the socialist tradition could not refuse to countenance. Hence, policy was concentrated for two years on reviving the economy, lowering the retirement age and slightly reducing statutory working hours (from forty to thirty-nine hours). Training was primarily mentioned in the light of the need to adapt workers to a new technological civilization.

It was in the same spirit that Jean-Pierre Chevènement, who became Minister of Education in 1984, announced that steps were to be taken so that 80% of an age-group would reach *baccalauréat* level by the year 2000.[2] It was thus hoped that, by raising the educational level of young people on the average, their difficulties in gaining access to the labour market would be reduced.

From 1986 to 1991, the rise in the growth rates of the French economy reduced unemployment and moderated the intensity of the debate on its causes. However, competition from low-wage countries came to be increasingly mentioned, while the debate on education became more animated, with teachers complaining about their working conditions, their salaries and budgetary restrictions and public opinion criticizing the cumbersome and ineffective nature of a bureaucratic education system.

Lastly, we come to the most recent period (1991-95), but the saga has not

ended. The 1991 recession, which was the harshest the French economy had experienced since before the war, increased unemployment, especially among young people and even among young diploma-holders. French society became frightened. The idea that the cost of unskilled labour had to be reduced began to gain ground, but the Balladur government had to face a revolt by students and secondary-school pupils when it proposed lower-wage work contracts (*contrats d'insertion professionnelle* [vocational entry contracts]) designed to facilitate the employment of young people. The under 25-year-olds did not regard themselves as second-class citizens.

During this time, the actions launched by the government, industry or the education system progressed, in that a vocational *baccalauréat* was created alongside the classical and technology *baccalauréats,* and the 80% target was attained several years in advance of the scheduled date, apprenticeship was promoted and alternation between schools and industry was fostered.

But the blight of unemployment lingers and gnaws away at French society. At the present time, the issues involved are all linked to exclusion, the break-up of society and the calling into question of the elites.

What lessons can be drawn from this overview which, in its brevity, is almost like a caricature?

1. The first thing that has to be noted is that French society has never been in agreement on the causes of unemployment. Opinion-makers have confined themselves to giving simple—and hence over-simplified—explanations such as: 'The cause of unemployment is . . .'. This intellectual shortcoming has had serious consequences since, in a democracy, political authority cannot readily take action if it does not have adequate support.
2. The second thing is that it was necessary to wait for nearly twenty years from the time unemployment first appeared before any notice started being taken of the people who mentioned the cost of labour as a root cause in classical unemployment. Indeed, French ideological traditions find it difficult to acknowledge the existence of a labour market, since they are influenced either by Marxism or by social Catholicism. In the case of Marxism, a worker's earnings are primarily the outcome of the power relations established between employer and employees, whereas in the case of social Catholicism, workers are expected to receive a fair wage that will enable them to bring up their families. In neither of these two cases is any reference made to the possible influence of labour costs for a category of individuals in relation to the volume of employment they are offered.
3. Owing to their lack of conviction regarding the root causes of unemployment, successive French governments, which have moreover been subjected to severe budgetary constraints, have kept on increasing the number of short-term employment plans, but have allocated only limited resources to each measure taken. They behaved implicitly as if the message they were seeking to convey to the population was: 'We have tried everything and hence nobody can hold our failure against us'.

It was necessary to recall this context before putting forward the diagnosis that an economist can make of unemployment in France.

Elements of a diagnosis

Compared with the history of unemployment as personally experienced by contemporaries, what is the diagnosis which should, in my opinion, be made by an economist who stands back and analyses the facts in all their complexity?

For the sake of concision, I shall attempt to make this diagnosis by simplifying and eliminating the many supporting observations. To start with, there are three essential observations:

1. The labour services that different individuals supply are not homogeneous. They are divided into types of skills that are a combination of knowledge, know-how and attitudes, with the last-mentioned component often being the most important.
2. In the French economy, a legally performed labour service involves two distinct costs: *a cost to the user* covering the gross salary, the employers' charges and the monetary equivalent of the different constraints bound up with the job (it is this price that employers take into account when making a job offer); and *a cost to the worker* that is equal to the difference between the gross wage and the charges, taxes and costs that the worker has to bear (it is this price that individuals consider when applying for a job). As everyone knows, the difference between these two prices is considerable. Employers and workers also take into account the possibilities for moonlighting, with its ensuing savings and risks.
3. Substitution between goods and services on the basis of prices is a key feature in the functioning of any economy. Depending on the case, the speed at which such substitution occurs may vary. Instances can be observed of substitution between labour services involving different types of skills, between labour and capital investment, between work performed on national territory and imports, between the production of a commodity or the decision not to produce it. There is no idea more false than believing in the existence of a volume of labour that is associated with a given level of national production, regardless of the prices involved. There is no Planck's constant in economics.

These preliminary observations having been made, there are three economic linkages that have been at the root of unemployment in France:

1. When there is a change in the job-supply-and-demand curves for each skill, the adaptation does not take place immediately. This accounts for the emergence of transitory *frictional unemployment* whose duration depends on the functioning of the labour market and the attitudes of the economic operators.
2. In cases where labour costs are rigid, two very different kinds of unemployment can develop: *Keynesian or cyclical unemployment* due to the fact that employers do not recruit because the resulting additional production would not find a market (this kind of unemployment, which varies over time,

affects a wide variety of skills; the policy which usually makes it possible to combat it consists in stimulating demand, a highly agreeable medicine even if it generates untoward effects); and *classical* or *structural employment* due to the fact that for *some* skills, the cost of labour is such that recruitment generates production where sales do not cover the additional costs (this form of unemployment, which results from both the price rigidity of the factors of production and their relative levels, primarily affects lower skills for reasons that will be mentioned below). Classical unemployment can also result from individual attitudes in cases where workers consider it more worthwhile to opt for a combination of unemployment benefits and leisure or unemployment benefits and moonlighting rather than for a combination of a wage and legal employment.

3. A distorted linkage may become established between Keynesian unemployment and classical unemployment: people losing their jobs following a recession may discover that their skills were mainly connected with the enterprise which employed them and that these skills are much lower in the eyes of other employers. They accordingly find themselves in a situation of classical unemployment if they seek to obtain a wage comparable to that which they previously earned.

Through the intermediary of these linkages, other factors that are not exclusively economic naturally have their part to play:

— technological progress which, with the emergence of information technologies, is changing the supply curves for a wide range of skills and, if the labour markets were flexible, would generate variations in the relative wage levels for different skills;
— international competition, which brings into contact countries where individual skill distribution patterns are different; for instance, the existence in the Third World of large numbers of unskilled workers and (in percentage terms) few skilled workers broadens the spread of labour costs per skill. In the developed countries, this would ensure that there would be no classical unemployment;
— governmental macroeconomic policies (especially monetary policies), which have an impact on the levels of economic activity and exchange rates.

Apart from its frictional component, unemployment in France is thus a mixture of cyclical unemployment accentuated by the monetary policy adopted from 1992 to 1994 and of classical unemployment due to the relative level of labour costs for lower skills and the limited interest which some of the unemployed show in obtaining legal employment.

Neither the purported absence of needs nor the labour supply of women and immigrants, nor poor growth as such, would generate unemployment if a suitable macroeconomic policy were adopted and if labour costs for the different skills matched supply and demand.

The most popular way of combating unemployment—by reducing fixed-wage working hours and hence increasing hourly labour costs—would, in the

medium-term, have effects opposite to those sought, because the rationing of scarce labour would, in the first instance, reduce the production possibilities and thus the job opportunities for workers with lower skills.

On the contrary, an effective policy should be a mix of a monetary policy offering greater incentives, a change in the basis of assessment for social charges, a counterpart requirement in return for unemployment benefits, the phasing-out of useless labour market formalities, and training measures.

Unfortunately, this diagnosis does not elicit the support of a sufficient proportion of French society for many different reasons that are bound up at one and the same time with:

— difficulty in understanding;
— the ideological models described in the first part of this article; and
— the strategies of social groups which fear that badly targeted reforms will be unfavourable to them.

However, on the basis of this diagnosis, it is possible to speculate about the lines of approach to an educational policy that would take employment problems into account.

What educational policy for which lines of approach?

Two preliminary remarks are essential to avoid any misunderstanding:

The education system is not responsible for the rise in unemployment in France. This rise is due to a refusal of French society as a whole to adapt itself to a different economic and technical context. While the instruments adopted in the 1950s in the name of security and equality gave rise to increasingly untoward effects, French society refused to change them in order to pursue the same social values more effectively in the new environment. It is this conservatism that is excluding individuals with lower skills from the labour market and which is confronting teachers, employers and workers alike with formidable difficulties: teachers because they have to reorient their action in order to enable their pupils to secure employment; employers because they are obliged to keep watch over the profitability of their enterprises; and workers because they have to make sure that they upgrade their skills.

It is not the purpose of the education system to train workers. Its social mission is to give men and women better opportunities for fulfilling themselves as human beings. This is a more far-reaching objective. However, in a difficult labour market situation, the ability to find work (or to find other work in the case of losing one's job) emerges as an almost necessary condition for full personal development. It should therefore be taken into account in educational policies, without of course detracting from the real objective of these policies.

These preliminaries having been outlined, the French experience draws attention to the various considerations described below.

TRAINING AND SKILLS

From the standpoint of combating unemployment, training should set out to change people's skills by enabling them to move out of categories where skills abound worldwide to categories where skills are harder to find on the same worldwide scale. This proposition has to be supplemented by two points of clarification:

1. Skills cannot be reduced merely to knowledge or to a combination of knowledge and know-how. They imply an attitude blending initiative and the ability to co-operate. It is this attitude which, along with knowledge and know-how, makes it possible, among other things, for individuals to adapt to changes in the technical, economic and social environment.
2. Skills are not equivalent to the qualifications often associated in France with the possession of a diploma, which primarily reflects knowledge, in some instances know-how and, to a lesser extent, an attitude.

As far as practicable, educational policies should therefore foster the gradual acquisition of creative and co-operative attitudes.

SOCIAL POSITION AND DIPLOMA LEVELS

Over the past two centuries French society has equated—especially in the civil service—social position with the level of the diploma obtained at the end of adolescence. This correlation between the two is now falling apart. Companies increasingly give preference to skills and there are grounds for thinking that the correlation between wages and diploma levels will in future be less evident for individuals as a whole.

Despite this long-term trend, diplomas are still very much sought after by individuals. A real paradox with regard to diplomas can be seen to be created by the following pattern:

- When they receive an influx of applications, employers use the diploma as an indicator of skills in the case of a first job. Obtaining a diploma accordingly enhances the likelihood of being recruited. This assumption is consistent with the statistics, which show that the unemployment rate falls with the level of the diploma. Once they have been recruited, people have good prospects of acquiring skills on the job. The initial advantage therefore tends to continue even if employers no longer attach importance to the diploma after a few years of job-related experience.
- In these circumstances, young people seek a diploma in order to be able to emit an 'audible' signal at the beginning of their careers and not to be eliminated prior to any interview.

TRADITIONAL DIPLOMA COURSES AND RE-ENTRY INTO THE WORK CIRCUIT

The more the percentage of diploma-holders rises, the more the relative situation of non-diploma holders worsens. The psychological, family and financial difficulties experienced during adolescence therefore tend to represent more severe handicaps than in the past, even though they concern a decreasing proportion of the population. The individuals involved often rebel against school. Bertrand Schwartz has clearly shown that we can only hope to instruct and educate them by tackling at one and the same time their employment, training and social integration problems, which is impossible in traditional diploma courses and entails specific approaches.

SCHOOLS AND EMPLOYERS

The importance of securing employment for young people has compelled France to progressively rethink the relationship between schools and employers. Many forms of co-operation have been developed at the local level between technical establishments, universities and the *grandes écoles,* on the one hand, and industrial and business leaders, on the other. At the national level, new laws have been enacted to promote sandwich courses and apprenticeship.

However, in the absence of a tradition as in Germany, there are many obstacles to be overcome. Apart from the legal difficulties involved (type of work contract, social coverage, etc.), French employers are not always organized to receive large numbers of apprentices in an efficient manner.

Furthermore, workers who wish to continue their studies can now obtain validation of their work experience enabling them to have direct access to the preparation of certain diplomas.

VOCATIONAL TRAINING AND ENTRY INTO WORKING LIFE

One debate that is constantly recurring concerns the broadening of pre-service vocational training: some people argue that the more this training is targeted the more it facilitates entry into working life, while others maintain that the broader it is, the more workers can adapt themselves to changes in production structures. It seems that the path to be followed is more often than not midway between the two, since too narrow a training makes people prisoners of a particular firm or sector, while too broad a training may handicap them in relation to other applicants when they are seeking their first job. Training geared towards some degree of versatility within the framework of a clearly-defined field may prove to be a compromise between professionalism and adaptability.

The problems that have just been raised have, however, to be resituated in a broader context. Adaptation to the labour market is not the only challenge facing the French education system at the close of this century. Mention can be made of several others, which have, in fact, been clearly identified: these include economic

and cultural globalization, the emergence of technologies giving rise to a new information society, the development of multidisciplinary and systemic approaches, the broadening scope of questions on ethical issues in the wake of scientific and technical progress, and so on.

This realization entails resituating relations between education and employment in an overall view of things. Unemployment is not due to economic and social inadequacies. It stems from a confusion of linkages which give it different forms but, *a contrario,* education in its life-enhancing ambition of shaping men and women must adapt itself to the technical, economic and social changes which are altering the conditions of employment in the present-day world.

Notes

1. *Rapport de la Commission Emploi et relations du travail du VIIIe Plan* [Report of the Employment and Working Relations Commission of the Seventh Plan], Paris, La Documentation française, 1980.
2. Meaning that 60-65% of an age-group would obtain the *baccalauréat* compared to 28% in 1985.

PIAGET

AND EDUCATION

FOREWORD

On the occasion of the centenary of the birth of Jean Piaget, UNESCO:IBE has decided to devote the open file of this edition of *Prospects* to 'Piaget and education'. We were asked to produce it and to ensure that the geographical diversity of researchers working with Piagetian theory was represented as fully as possible. This edition is not intended simply as a commemoration of Piaget's work. We felt it would be closer to the spirit in which his work developed to show not only the fundamentals of genetic epistemology and genetic psychology but also the debates involving other theoretical contributions which take them further forward, and also the changes resulting from a reappraisal of concepts in the field of education. This open file follows up the general directions currently being taken by education science research linked to genetic epistemology and genetic psychology. These directions also draw on scientific advances in cognitive psychology, intercultural psychosociology and the didactics of the disciplines. We have thus divided the volume into three sections: 1. Piaget, the mechanisms of development and school learning; 2. Piaget and the social aspects of knowledge; 3. Piaget and didactics.

In the first section Lauren Resnick opens the debate on the mechanisms of development and learning with an attempt to integrate theories dealing with biological constraints on the one hand and situated cognition on the other. Marcel Crahay describes various stages in the pedagogical interpretation of Piagetian theory with reference to the classic dilemma regarding education: learning to memorize or learning to think? He concludes by looking at certain key ideas of genetic psychology in the light of certain aspects of recent developments in Anglo-Saxon cognitive psychology, in particular in relation to problem-solving.

Ludmilla Obukhova raises the question, always a crucial one, of the relation between learning and development by showing how the child constructs the tool-mediators proposed by the adult and how those tools become internalized operations.

Constance Kamii emphasizes development. School learning is possible only if we update our knowledge of development. Thus, in teaching arithmetic, knowledge must first and foremost emerge from previously constructed

logico-mathematic knowledge. For her, Piagetian theory should be applied rather than questioned.

Like Constance Kamii, Takehisa Takizawa seeks more or less direct inspiration in Piaget, in order to provide a scientific basis for teaching. He shows how genetic psychology has been applied in Japan, following a historical approach.

The ascendancy of culture over knowledge is, however, clearly asserted in the three chapters of the second section. Mohamed Lahlou raises the problems of school integration in the political and social context of the previously colonized countries, the problems of the status of the languages taught at school and their relation to first languages, and the problem of the first language as a vehicle for academic knowledge. He stresses the key role of social interactions in the teaching-learning process and highlights the importance of the child's adaptive activity in these interactions.

Emilia Ferreiro tackles the acquisition of cultural objects, such as a written language, and wonders if Piagetian theory can account for the mastery of all objects of knowledge. She concludes that it is a 'general theory of the processes whereby knowledge is built up'.

Eduardo Martí is closer to a concept of the heterogeneity of knowledge based on the content of specific branches, examining the links between certain fundamental concepts of genetic psychology and those developed by another great psychologist, born in the same year as Piaget, Lev Vygotsky. Piagetian constructivism is seen as an indispensable basis for any consideration of the teaching-learning process and as being linked with some of the tenets of Vygotskian socio-cultural theory.

The chapters of the third section also take on board the socio-cultural dimension of knowledge, but situate knowledge in the field of didactics, defined as a system designed to teach a body of knowledge that exists culturally.

Gisèle Lemoyne devotes her chapter to the conversion of what a child knows into what a pupil learns, and vice versa. She applies aspects of genetic epistemology to the didactics of mathematics, describes the body of problems covered by each of the two fields and presents experimental findings which attest to the usefulness of combining them.

Gérard Vergnaud brings the open file to a close by suggesting that a certain distance should be kept both from the didactics of mathematics and from genetic epistemology. He identifies issues that emerge from a comparison of the two fields, namely: knowledge as an adaptive process, the development of general structures and specific conceptualizations, situated action and underlying conceptualizations.

We for our part open this discussion of the pedagogical developments of genetic psychology with a presentation of the work done in Geneva, which is of course grounded in genetic psychology but which also extends it by studying the interactions between knowledge and situation, in which we stress the role of didactic situations in the structuring of knowledge in pupils.

Two key points emerge from most of the chapters. The first concerns the

concept of a *generally applicable overall structure,* which is an epistemological obstacle for pedagogy. In fact, almost all the contributors emphasize the heterogeneity of knowledge, whether in respect of cognitive processes, the specific features of different socio-cultural contexts or the specificity of the situated knowledge. The second leitmotif concerns precisely the importance of the specific contents on which and within which knowledge is built up in the school context.

MADELON SAADA-ROBERT
JEAN BRUN

TRANSFORMATIONS OF SCHOOL KNOWLEDGE: THE CONTRIBUTIONS AND EXTENSIONS OF GENETIC PSYCHOLOGY

Madelon Saada-Robert and Jean Brun

From the construction of knowledge to transformations of situated knowledge in the child

Four major periods may be distinguished in Piaget's work on the psychological development of thought in the child. The first period lays the foundations of the child's reasoning, characterized not yet by logical structures but already by authentic forms that cannot be reduced to those of the adult (*La représentation du monde chez l'enfant,* 1926). The second period, beginning with *La naissance de l'intelligence chez l'enfant* (1936) and culminating with *La genèse des structures logique*

Madelon Saada-Robert (Switzerland)
Madelon Saada-Robert has a doctorate in genetic psychology and was for many years research and teaching assistant to Professors Piaget and Inhelder at the University of Geneva and the Centre for Genetic Epistemology where she worked under the direction of Jean Piaget. She went on to specialize in representation linked to the functioning of contextualized knowledge, in particular with pre-school children. She is currently senior lecturer and researcher in education sciences at Geneva University, working on the learning of written language in schools.

Jean Brun (France)
Professor of the didactics of mathematics in the Faculty of Psychology and Educational Sciences of the University of Geneva. Jean Brun holds a doctorate in psychology from the University of Lyon II; his thesis on the relations between intellectual development and the introduction of the new mathematics in the early 1970s explores the effects of that teaching. His present research continues to focus on those relations, now taking into account the teaching situation itself at the elementary schooling level (6 to 12 years).

élémentaires (1959) [with Inhelder], defines the characteristics of the logico-mathematical structures peculiar to the various stages of cognitive development, while demonstrating the fact that operations are rooted in actions. The third period effects a de-centration of the structures of the subject towards the pole of the object, studying its role in the construction of knowledge (*Les explications causales,* 1971; *La prise de conscience,* 1974). Then the fourth period reverts to the central question of the mechanisms of development, linking the assimilation-accommodation relations with the study of *L'équilibration des structures cognitives* (1975) and *Recherches sur l'abstraction réfléchissante* (1977).

That final period, extending to study of more psychological themes, or at least themes not strictly amenable to formalization on the basis of logico-mathematical models (*Recherches sur la généralisation,* 1978; *Recherches sur les correspondances,* 1980; *Vers une logique des significations,* 1987 [with García]), interests us more particularly. It poses the basic problem of the functioning of knowledge, an issue elaborated upon and expanded by the studies on the links between deep structures and situational knowledge, and particularly on the role of representations (Inhelder et al., 1992). Because of their impact on an understanding of the cognitive function in pupils in classroom situations, we shall initially give a brief account of the two main subjects of these studies (representations and their microgenesis), after showing on which main tendencies in genetic psychology they are based.

The constructivist and interactionist foundations of genetic psychology

Even if these studies challenge the idea of the universality of structures which is implicit in their own definition, and hence the idea of their quasi-automatic transfer to specific situations, they are nonetheless based on the two underpinnings of genetic psychology: constructivism and interactionism. First, they take constructivism further by extending it into learning contexts. For these studies show that, in its functioning, even acquired knowledge is not simply applied to a situation but reconstructed according to the structure of the situation. The acquisition of any new knowledge therefore has to bring into play earlier knowledge in specific situations through which it is transformed, differentiated, and set against the realities of the situation, and finally results in new abilities. They then take interactionism further by emphasizing the structuring role of the situation, which must be understood in both its social and its cognitive aspects.

The construction of knowledge and the role of interaction

Four mechanisms are central to the cognitive function in genetic psychology. The studies on situational representations review these, stressing in each case the structuring role of the situations in which they function. They are the relations between

assimilation and accommodation, realization, reflexive abstraction, and equilibration.

The role of accommodation in relation to assimilation. By concentrating on the contribution of accommodation, which is involved in any relation of the subject with the environment for the purpose of gleaning relevant information from it, the studies on representations overturn the importance given by Piagetian theory to assimilation as a prime factor and a reproducing, generalizing propensity. They see accommodation as pivotal to receptiveness to the environment, with its properties and its constraints, which indicate to subjects the direction in which they will be able to adjust to it.

Consciousness and the role of the external situation. This is the only mechanism concerning for which Piaget posits a movement from the outside inward, from the periphery (specific situation) to the centres (conceptualization of information by the subject, and parallel constitution of the properties of objects). Consciousness is essentially defined as conceptualization of the results of the action and conceptualization of the properties of the object. This definition seems to us to be too restrictive, for there are items of information derived from the situation which may remain relatively implicit, without being exposed to a level of explication that formalizes them.

Simple abstraction compared with reflexive abstraction. Reflexive abstraction can briefly be said to enable the subject to reconsider (in reflection), in a coherent whole defining a higher structural level, skills which have evolved in relative isolation and have been derived from a more elementary level. By contrast, simple abstraction is that which enables subjects to derive information from the situation: either specific properties of objects through their actions, or the results of those actions on the situation. For Piaget, such abstraction serves as 'food' for reflexive abstraction. These abilities, built up in direct relation with the situation, will be consolidated in different situations (this involves reflectiveness, the movement of abstraction from one level towards a more complex level). In this mechanism, Piaget's analysis emphasizes reflexive abstraction and more particularly the reconstruction of structures, once their components have been drawn from the more elementary level. A closer examination of the situational constructs of subjects nevertheless shows that simple abstraction, because of the significance attributed by the subject to the various parameters of the situation, and owing to its comparison with reality, plays a greater constructive role than Piaget's analysis suggests. Whenever children, or pupils in a classroom situation, cannot use a piece of knowledge as such to solve a problem, and this is normally the case whenever learning takes place, they must 'make the situation talk', i.e. give it a meaning, in order to derive from it new factors pointing to where they must seek a possible new solution, by constructing new means of achieving that goal (or a new combination of acquired skills).

Equilibration and the role of the beta stage. When a disruption occurs in a chain of reasoning and in the associated procedure, whether stemming from the situation (when behaviour meets with an obstacle, for example, or when new properties must be taken into account) or from representations of the subject (such

as two conflicting units of knowledge), three reactions may occur in the subject, which are graded in three stages leading to the final overcoming of the disequilibrium caused by the disruption. In the first stage, alpha, the subjects annul the disruption by avoiding it, thereby recovering the initial equilibrium. In the second stage, beta, the disruption is acknowledged but the subjects cannot yet overcome it. They will accept it but only as a special case, or as an exception, which does not fundamentally alter their system of thought. In this case, they remain in a position of relative disequilibrium. In the third stage, gamma, subjects will not only acknowledge the disruption but will incorporate it in their system of thought by making the latter more sophisticated. The disruption will then be overcome and a new equilibrium will be struck that is steadier than that of the alpha stage. While Piaget mainly concerned himself with finding the conditions for overcoming the disruptions (the gamma stage), recent studies seek rather to show the importance of the beta stage, that which enables subjects to be receptive to the environment by allowing for possible disruptions. Even if they have not yet overcome them, they must be able to consider, read and accept them, if only as special cases. And it is partly through this reading of new, hitherto 'hidden' properties of the situation that subjects will find the path towards overcoming the disruption.

The study of representations and the microgenesis of problem-solving

The team investigating the cognitive function in genetic psychology (Inhelder et al., 1992) focused on the study and microgenesis of representations.

Representations are examined therein as the interface of earlier knowing[1] and the properties of the situation. It is the locus of formation of situated skills and action procedures whereby the child will resolve the problems posed in the situation. The situated cognitive representations are defined by the relation between the deepest units of functioning—*context-free knowing* in long-term memory: invariant structures, schemas, operations, specific knowledge such as scripts, outlines, etc.—and more external units: *sequential procedures* dependent on the intentions and projects of the subject. Between the two types of unit, an interface is constructed through the representations situated at the working memory level.

Any situation is defined by the subject in terms of a chain of goals and subgoals, and in terms of problem-solving sequences, even when appropriate solutions are not found. In this context, problem-solving microgenesis may be regarded as a change of meanings for the subject; a given action, defined as a context-free schema, may be specified as routine (when functionally attached to the objects of the situation irrespective of the goal pursued), as primitive (when regarded as a key to the problem) or as a procedure (when composed of key primitives). In the case of a known problem, or of a new but similar problem, these three forms of action do not all necessarily enter the picture. A correct procedure used as routine may be specified in the course of a top-down process. In the case of an entirely new problem, the three forms of action appear in the prob-

lem-solving microgenesis. Some relevant actions may be seen as routine in the course of an bottom-up process; others are specified as key primitives, and verification then amounts to building them into procedure.

Microgenesis study, as undertaken by the Geneva team of researchers on the cognitive functioning, does not seek to look further into what is known of macrogenesis, particularly as developed in genetic psychology. Nor does it set out to reconstitute macrogenesis on a smaller scale or to find the famous three stages in the instant when the problem is solved. Microgenesis study, rather, yields a better understanding of the mechanisms of change, previously seen either in general terms of reflexive abstraction or majoring equilibration with its three levels—alpha, beta and gamma—or in terms of fine transitions and cognitive conflict by Inhelder and her collaborators (Inhelder, Sinclair & Bovet, 1974). These change mechanisms are essentially understood as changes of signification and changes of monitoring by the subject. Microgenesis study also seeks a grasp of how relations operate between the connected components. It thereby offers an alternative solution to the connectionist models which could be applied to cases where the components have already been automated and which would explain how they are activated, but which certainly do not explain how the relations between components are constructed through their functioning in context. Microgenesis study also shows that the basic roots of the structures are not constructed through isolated filiation but, rather, within conceptual fields (Vergnaud, 1985), through relations of alternating interdependence and reciprocity. This is possible because these microgenesis studies have deliberately focused on the functional aspects of the formation and activation of knowledge. Directing research towards the functional, rather than the exclusively structural, aspects amounts to an acknowledgement of the role of the situation considered in an interactive relation with the subject. Thus, in the dialectic of Piaget's construction-interaction poles, we incline more towards the interaction pole, and by studying interaction we gain a better idea of the mechanisms of construction; which brings us to the linkage between learning and development.

Solving school problems

The school lies within a network of definite, albeit shifting, socio-institutional significations. The pupil, the teacher and the knowledge are its didactical protagonists. In this setting, the problem-solving situations actualizing the teaching/learning relationship are complex. They are governed by a didactical contract that will enable pupils to attribute particular values to a part of their knowing concerned by the problem, and that part they will transform into situated knowledge.

Recent research in genetic psychology on representations has not been concerned with problem-solving in school situations. However, in an educational system, focusing on the teaching/learning linkage seems to us to raise a number of specific problems. Here we consider the problem of what are termed complex situations: broad situations that permit the *integration* of knowledge regarding specific contents. The merit of such situations is threefold.

Pupils, firstly, are afforded a specific entry into the open problem, a distinctive path to be followed in solving it, and their own monitoring of goals and components. Activation of the relevant knowing and construction of the situated knowledge will thus take place through instances of bona fide microgenesis (Saada-Robert, 1995).

Then for teachers, complex situations ensure the necessary embedding of teaching in the pupils' representations, to the extent that the teacher observes the procedures involved and associates them with the conceptual field underlying each situation. Through such observations and the appropriate regulations, the necessary differentiation for linking teaching with the various forms of learning will be able to proceed.

Finally, for researchers, the relevance of complex school situations lies in the fact that their variables, from a social and cognitive point of view, are closely linked, and that the time spent on problem-solving by the pupil, with or without real interactions, permits study of the microgenesis of situated knowledge.

The transformation of situated knowledge

The development of research in genetic psychology increasingly focuses on attempts to understand how knowledge is constructed (see Inhelder et al., 1992). It is stressed that knowledge acquired by a subject does not in fact contain, of itself, its conditions of application. The situation plays a role in its utilization and hence in its construction. Psychological constructivism may accordingly be regarded as fundamentally interactionist.

Research in didactics, on account of its object, calls for more study of the role of situations. A teaching situation is overdetermined by the knowledge with which it is concerned and by the institution responsible for transmitting it. The representations developed by the pupils in these circumstances bear the imprint of this overdetermination, so that, in a teaching context, *didactics has a hand in defining what is representable.* Didactics might then be thought to be only a curb on, or even an obstacle to, the natural development of the subjects' knowledge, and it might consequently be proposed to reorganize didactics around this natural development, seen as the only vehicle of the child's 'true' potential. Development would be normative to the extent of ensuring the reappropriation, by individuals, of instituted knowledge. Experience shows that didactics are resistant—that they cannot be refashioned at will in line with discoveries regarding child development, even though such discoveries have a very important part to play in teaching. Didactics need to be regarded as a subject of study *per se* (Brun, 1994). Without entering here into a general review of the didactic phenomenon, which no doubt pertains to anthropology (Chevallard, 1991, 1992, 1994), let us merely consider its effects on the concept of situation as a locus of cognitive interactions. We shall do so from the standpoint of mathematics didactic.

We formulate the following problem: to what situations may the meeting of a subject with a formal and, to that person, new knowledge correspond? What

are the conditions for such a meeting? We do say 'meeting' in reference to Mercier (1994). Genetic psychology teaches us that such subjects have forged knowing through their own development mechanisms (equilibration, simple and reflexive abstractions, consciousness) in contact with a variety of situations which, in this case, refer to their overall experience. Subjects thus construct themselves progressively and structure themselves cognitively. In a situation, in the broad sense of the term, they utilize their knowing and construct fresh knowing. Microgenetic studies have pinpointed the processes whereby the child makes new discoveries (see Inhelder et al., 1992).

This developmental process meets with didactics at the point in time when social intention is brought to bear on it, namely, the intention of teaching knowledge culture hands on from generation to generation.

We then enter a class of specific situations which will be defined as didactic, provided that they combine developing knowledge, an intention to teach and a knowledge 'already there' (Rouchier, 1991). The interactions between these three factors define the 'didactic situation', the model form of the teaching situation. In comparison with the interactionist pattern 'subject x situation', the distinctiveness of the didactic situation therefore requires clarification.

Knowledge and didactic situations

The study of the actions adopted by pupils when faced with a mathematical problem, in a teaching situation, also requires the significance of such actions to be set within the characteristics of that situation. Those characteristics are epistemological and cognitive, but also didactic. The situation is charged with intentions and expectations on the part of the teacher, besides being located within an educational progression; these factors organize and regulate the representation that pupils forge of the mathematical problem presented to them.

At this stage of reflection, we are faced with at least two major alternatives in the choice of an approach to research: that which consists in declining to espouse this pattern and pushing to its limits the hypothesis of the autonomous reconstruction of knowledge by the subject (radical constructivism); and that which involves querying the conditions under which there may be a meeting between knowledge already present, formed culturally, and the organized patterns of knowing constructed by the epistemic subject. Studying a didactic situation means studying just such conditions (Mercier, 1994). The interconnection, via situations, between individual knowing and instituted knowledge is the core of didactic issues.

Various experimental studies conclude that there is in fact no natural transition from knowing produced by psychogenesis to instituted knowledge. Berthelot and Salin (1992) have shown, for example, that the spatial notions of pupils are not simply converted, by the mere fact of cognitive development, into geometrical knowledge (they may sometimes even be a hindrance); nor does the description of instituted knowledge suffice on its own to find an echo in the pupil's spatial knowing, which is nevertheless necessary to the teaching of such

concepts. Connecting knowledge with individual knowing is an ongoing didactic problem (Conne, 1992); its solution depends on, *inter alia,* the didactic creation of objects to be taught, a task of transposition that cannot be performed without that experimental epistemology constituted by research in didactics.

Brousseau (1983) touches on that task when he writes (our emphasis):

> We shall therefore assume that *the constitution of meaning,* as we understand it, implies a constant interaction of the pupils with problem situations, a dialectical interaction (for subjects anticipate and finalize their actions) in which they engage previous knowledge before reviewing, modifying, completing or discarding it to form new conceptions. *Didactics is, as it happens, chiefly concerned with studying the conditions that must be fulfilled by the situations or the problems proposed to the pupil* in order to encourage the emergence, functioning and rejection of those conceptions (p. 172). *[Translation.]*

The meaning of a mathematical knowledge depends first on the state of organization of the subject's knowledge (see Vergnaud, 1991). That meaning is also determined by a set of cultural problems for which knowledge (the idea of measurement, for example) is required for the discovery of a solution. Brousseau has studied a great many situations corresponding to such sets of problems; he calls them fundamental situations. Now, the notion of a 'fundamental situation' brings us back to the epistemological characteristics of didactic situations. The choice and organization of such situations, in the course of teaching, is part of an experimental epistemology that asks the question: 'How is knowledge transformed from its origins to the point at which it is taught?'

The transformation of knowledge for teaching purposes is what occurs in the process of *didactic transposition* (Chevallard, 1991). Behind this transformation are to be found the professional mathematicians who themselves engage in the work of 'decontextualizing' and depersonalizing their knowledge, as yet private, in order to make it communicable to their peers and beyond. Taken as part of the educational task and marked out 'to be taught', such a context-free knowledge must come into contact with new persons: the pupils. For that meeting to be possible, however, the knowledge must be put back into context and repersonalized, the point being that pupils are supposed *themselves to produce the modification of their knowing,* as genetic psychology tells us. Knowledge cannot therefore be passed to them directly. It is by means of new and, this time, didactic context-building that pupils become capable of modifying their knowledge themselves. At the end of the process, if the meeting has actually occurred, this personal knowing will still need to be converted into instituted knowledge.

While the personal modification of pupils' knowing is the essential characteristic of the didactic situation, the latter also makes it necessary to recognize a status for the knowledge brought to bear by the pupil in the situation. Actual experience invariably confirms this when attempts are made to reduce teaching to a series of such learning operations in situations. That necessity stems from both pupil and teacher, or more exactly from the relationship which is theirs. They need mutually to acknowledge and take note of learning, if only to be able subse-

quently to re-use it in the course of the teaching process. This institutionalization of learning is a necessary condition for a lasting didactic relationship (and not just some form of compulsory lip service to the institution). Hence, some work has to be done on context-stripping to permit cultural recognition of the knowledge involved in the personal relations between pupil and situation, and to make it possible to communicate this knowledge within the didactic relationship, just as mathematicians do *vis-à-vis* their scientific community by means of the transposition process just described.

A shift is therefore effected in relation to genetic psychology. It is the *meaning of knowledge* that becomes the issue at stake in mathematics teaching and is studied by the researcher. This shift is a particularly interesting variant on the whole cognitive issue. For didactics, to consider instituted knowledge is not to overlook the knowing of the developing subject—knowing whose profoundly epistemological significance has, we should never forget, been known since Piaget. It is rather to look beyond, or before, such knowing, as the case may be, on account of the fact that situations and practices will place the subject's knowing in a new relationship where it is possible to 'meet the need to know'. This in no way, as yet, prejudges the effective character of that meeting (Mercier, 1994).

The priority issue for the specialist in didactics is, then, the choice of situations representing a body of knowledge. Which are those situations which bring about a meeting with the necessity of knowing, over and above the will of the teacher serving as the vector of the teaching project? Let there be no mistake about the meaning of the expressions 'meeting the necessity of knowing' and 'situations reflecting a body of knowledge'; they do not refer to empiricist or Platonic positions conceiving of such knowledge as presented with the situation, or hidden in the situation. They refer to a necessity linked to 'practices', or 'games', which are social and which echo other practices and other games, in other places where knowledge is built up. So it is that bodies of knowledge are transformed. Choosing situations calls for experimental studies, and these are the concern of didactic research.

Problem-solving and didactic situations

One way of looking at the distinctiveness of the didactic situation is to try to understand its role in problem-solving processes. Problem-solving provides interesting food for thought in mathematics teaching to the extent that the mathematician's work is, essentially, problem-solving. Hence much effort goes into teaching in order to transpose to pupils this type of activity that pertains to the mathematician. Naturally, therefore, research on mathematics teaching has drawn upon the aims and methods of the psychologists who study problem-solving. The question is whether a problem area can be simply transferred from psychology to didactics. The studies of Schoenfeld (1985), for example, provide an excellent account of the general processes whereby pupils solving problems in mathematics use and shape their knowledge in the face of a problem presented to them.

We still, it would seem, have to understand the role played by the actual

situations in which pupils interact with the problem put to them, for that role is undoubtedly decisive in teaching. An analysis of the situation becomes essential in order to extend what is referred to as task analysis, seen as the sequel to the thought operations or processes needed for solving the problem. Situation analysis considers the various possible relationships of the pupil with the problem, in the light of the choices made in the prior organization of the teaching situation; and indeed, those choices determine possible relationships. From the standpoint of the pupils, this refers to their representation of the operational setting for their knowledge, to such processing as they feel able to perform, and to the means of checking the solutions they deduce from it.

What is the part played by this operational setting in the problem-solving process, and how does it evolve as the solving proceeds? This operational setting may be considered to serve as a means of monitoring the problem-solving.

The first type of monitoring is that which is internal to the subject. As Blanchet (1994) writes:

> Throughout the research the subject is attentive, checking and evaluating each element, confirming particular starting-points and questioning other points. It is a dialectic between action, representation and processing. It is the subject who oversees this dialectic, *even if not aware of all the factors influencing it,* and even if chance may also operate (p. 57, our emphasis.) *[Translation.]*

Among the factors influencing this cognitive monitoring, and with which research in didactics is concerned in that it acts experimentally upon them, are checking patterns derived this time from the situation: an *a priori* control, firstly, linked to the transposition process, together with the choice of problems and situations that refer to the social *practices* behind the production of mathematics. Knowledge is primarily, as formulated by Chevallard, *knowing in action:* 'Any knowledge', he wrote, 'is for the actor knowing in action. All knowing is social practice. Let us not be surprised then, if a particular mathematician wonders at our giving the status of knowledge to mathematics, which for him or her is primarily *doing*' (1994, p. 176). That is the starting-point, as we have seen, for the didactic transposition. The choices then made for the didactic staging of the situations are also means of acting upon the problem-solving process and constituting the meaning of the knowledge; they are tested at the time of the didactic creation of the subject-matter to be taught. Brousseau (1986) theorizes about this staging of situations by means of situations calling for action, formulation and validation, which tie in with the various functions of knowledge, and by means of various didactic phenomena associated, for example, with the epistemology of the teacher, or with the didactic contract (Brousseau, 1986; Schubauer-Leoni, 1986).

In our first experimental attempts at understanding these external types of monitoring said to act upon the pupils' representations and processing, we have managed to verify, by closely examining observation records, the part played by some didactic phenomena in the problem-solving process (Brun and Conne, 1990).

In distance-measuring situations, we observed how social measuring practices, such as the familiar use of a graduated ruler or of a double metre rule, interfered with the pupils' knowing of measurement in the problem-solving process. We were then able to understand how a 'breach of contract' in the situation confronting the pupils affected them by depriving them of their customary benchmarks in measuring exercises. That breach of contract influenced the problem-solving process, sometimes hindering and sometimes assisting the functioning of pupils' knowing of measurement. Indeed, it should not be supposed that these didactic phenomena do nothing but hamper an ostensibly *bona fide* problem-solving activity.

Didactics research prompts a rethinking of particular aspects of the general models describing the regular sequences of problem-solving mechanisms that operate in a pupil proficient in such problem-solving. Didactic phenomena may interfere with these mechanisms and complicate the model. We see this as emblematic of the conditions presented by the situation to this particular problem-solver, namely, the pupil interacting with a problem in class. This opens up a whole field of research which, with this choice of problem area, tends towards the construction of experimental didactic situations characterized by mathematical objects of knowledge, rather than towards drilling pupils in the hope of endowing them with general expertise in problem-solving.

Notes

1. From now on in this paper, we will use the concept of *knowing (connaisance)* when referring to the subject of our own deep knowledge, and the concept of *knowledge (savoir)* when referring either to the subject's situated knowledge when encountering the situation or to the instituted knowledge as it defines the structure of the situation itself. From this point of view, both situated and instituted knowledges reflect each other when the child masters the situation.

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SITUATED RATIONALISM: THE BIOLOGICAL AND CULTURAL FOUNDATIONS FOR LEARNING

Lauren B. Resnick

In this article, I examine the relations between two lines of thinking—each commanding increasing attention among psychologists and social scientists—that appear to be contradictory. The first, a position I term conceptual rationalism, seeks biological foundations for specific concepts that are central and, perhaps, universal in human development. The second, a position that has come to be known as situated cognition, argues that knowledge is acquired in and attuned to specific social and historical situations and that conceptual development can be understood only in terms of the situational contexts of action. I argue here that the rationalist and situationist views, far from being contradictory, share important epistemological assumptions and can—perhaps must—be combined to provide a theory of cognitive development and functioning. I develop a view of learning and development that I call 'situated rationalism', illustrate it with some examples from mathematics and science learning, and consider its implications for education.

Lauren B. Resnick (United States of America)
Professor of Psychology at the University of Pittsburgh, where she directs the prestigious Learning Research and Development Center. She is an internationally known scholar in the cognitive science of learning and instruction. Her recent research has focused on assessment, the nature and development of thinking abilities, and the relationship between school learning and everyday competence. She has served on the Commission on Behavioral and Social Sciences and on the Mathematical Sciences Education Board at the National Research Council. Recent publications include: *Education and learning to think* (1987) *and Learning in school and out (1987).*

Conceptual rationalism: biological preparedness for learning

In recent years, there has been a reassertion of interest in the biological basis of human learning and thinking (e.g. Gelman & Carey, 1991; Hirschfeld & Gelman, 1994). This new line of thinking grows out of research on language development, concept development in infancy and early childhood, and animal cognition and learning. The core proposal of those pursuing this line of thinking is that there exists a set of biological constraints on learning and cognitive development. This hypothesis leads to a search for evidence that certain aspects of knowledge, although learned in the sense that interaction with the environment over time is required, are nevertheless biologically preferred or 'prepared'. These highly specific prepared schemas or skeletal structures are the foundation for the development of mature knowledge by individuals. The argument is, roughly, that each species is specialized for certain forms of knowledge. This biologically preferred knowledge is tuned to the adaptive requirements of the species. It prepares the young of the species to enter productively into the situations they are most likely to encounter as they grow and to learn from behaving in those situations. Until recently it was most frequently employed as a theory of language acquisition. The argument for biological or 'hard-wired' structures that guide and constrain infants as they interpret their earliest experience is now being put forth for basic mathematical, physical and social concepts as well.

Following philosophical traditions, this can usefully be termed a rationalist position; it is reflected in the epistemologies of Plato and Kant, for example. Rationalists differ from associationists and other empiricists not only in postulating a biological basis for specific knowledge, but also in postulating wholes—sets of relationships that accumulate to something more than the sum of their parts—as the fundamental units of cognition. This is captured in the notion of a schema—an organizing 'design' that superimposes a structure on the pieces. Among the great rationalists who had something to say about education and learning were Wertheimer, for whom the organizing designs were perceptual gestalts, and Piaget, for whom the organizing designs were logical structures.

I call today's rationalists 'conceptual rationalists', because they are more interested in conceptual than in either perceptual or purely logical foundations for thinking. Conceptual rationalists claim that biological preparation is highly domain specific, that infants are biologically prepared to take advantage of very specific affording features of the environment. They believe that the preparedness of the species reaches beyond perceptions (supposedly minimally processed recognitions—cf. Neisser, 1976). They are interested in preparedness for reasoning about number and quantity; for concepts of causality; for notions of weightedness, movement and rigidity; and for basic psychological and social ideas. These are among the concepts that Piaget studied in his early work, but he later emphasized the much broader logico-deductive structures that he claimed underlie

individuals' developing capacities to reason and learn. Yet Piaget was certainly a rationalist in his argument that humans are biologically prepared to develop these particular structures of reasoning.

My inclusion of Piaget among the great rationalists may be surprising to some readers, because it is common to contrast Piaget's constructivist theory of cognitive development with more biologically determinist views. The famous debates between Piaget and Chomsky (Piattelli-Palmarini, 1980), for example, are frequently used to illustrate this distinction between 'inherent' language structures as laid out by Chomsky and Piaget's psychogenetic theory of conceptual development.

But, like Piaget, today's conceptual rationalists do not claim that hard-wired concepts simply mature. Rather, for the biological endowment to be realized, particular environmental conditions must be met. Prepared structures do not substitute for learning but rather make learning possible by constraining and guiding attention, so that, from among the many stimuli children encounter, they select for attention those that will support the formation of particular concepts. Conceptual rationalism is a theory that says children can teach themselves if the right kinds of affordances are present in their environments. Children appear to choose for themselves the kind of stimuli to attend to and to engage in forms of practice that eventually establish a stable and useful concept. Learning and development occur when individuals prepared for certain concepts encounter environments with the kinds of affordances they need to elaborate these prepared structures.

Today's conceptual rationalists, focused as they are on identifying the specific skeletal knowledge structures, may not use the language of constructivism. But their theories of self-teaching constrained by guiding skeletal structures are generally coherent with Piaget's constructivist stance. Like Piaget, the conceptual rationalists are relatively uninterested in individual differences or cultural variations in development. The prepared concepts they study are thought to be universal for the species. And, like Piaget, they give relatively little attention to social processes, although these are admitted as part of the environmental surround that allows children to elaborate their prepared schemas. Furthermore, like Piaget, conceptual rationalists are focused exclusively on individual learning and cognitive development. In their theories, the biologically prepared structures that they study are carried by each individual member of the species, and each individual must interact with the environment to produce personal elaborations of the prepared structures.

Situated cognition: socio-cultural preparedness

The rationalist focus on the biologically-endowed individual contrasts sharply with an alternative view that cognition and, therefore, learning are inherently social. The term 'situated cognition' has come to refer to a loose collection of theories and perspectives that propose a contextualized and social view of the nature of thinking and learning. Students of situated cognition take as a starting

point the distributed nature of cognitive activity—the fact that, under normal circumstances, mental activity involves social co-ordination. Getting a job done and figuring something out are almost always done in co-ordination with others. What makes an individual competent is not just what he or she knows, but also how his or her knowledge fits in with that of others with whom activity must be co-ordinated. Furthermore, activity is often shared with tools (e.g. Hutchins, 1991) and even with the everyday physical material about which people reason (Lave, 1988). There is, thus, a distribution of cognitive work not only among people but also between people and tools. Being competent means being able to use particular tools in particular ways.

Tools themselves embody a portion of the intelligence that is needed to accomplish tasks. The distributed nature of competent performance means that competence is highly situation specific. One must be good at behaving in a particular situation, with particular tools and with particular other people. The situated cognition perspective, then, tends to lead away from a search for general structures of knowledge and toward the study of specific environments for cognitive activity and the knowledge attuned to those environments. At the same time, it stresses the social nature of cognitive activity and cognitive development.

In the situated cognition view, the social invisibly pervades even situations that appear to consist of individuals engaged in private cognitive activity. Social construals of the situation (e.g. What are the rules of the game? Who is in charge? What are the stakes?) influence the nature and course of thinking. And the tools of thought (ranging from external memory devices and measuring instruments to tables of arithmetic conversions and dictionaries, thesauruses, and maps) embody a culture's intellectual history. Tools have theories built into them, and users accept these theories—albeit often unknowingly—when they use these tools. This point is made dramatically by Latour (1987) in his constructed account of the process of challenging a scientific conclusion. Like biologically prepared structures, the tools that one uses not only enable thought and intellectual progress but also constrain and limit the range of what can be thought. In these invisible ways, the history of a culture, an inherently social history, is carried into each individual act of cognition (Cole, 1985).

Theories, implicit and explicit, both enable and constrain thinking, just as physical tools do. This observation has become commonplace in cognitive science. What individuals reason about, the knowledge they bring to a cognitive task, provides the interpretive frames or schemas that allow reasoning and problem-solving to proceed. These beliefs, individuals' schemas for reasoning, are not purely individual constructions. Instead, they are heavily influenced by the kinds of beliefs and reasoning schemas available in the individuals' surrounding culture.

Not only theories but even ways of reasoning are themselves socially determined. Cognitive tools also include the forms of reasoning and argumentation that are accepted as normative in given cultures. Both Mead (1934) and Vygotsky (1978) proposed that mechanisms of thought are best conceived as internalizations of behaviours first engaged in externally, interacting with others. Mead

called thought a 'conversation with the generalized other', implying that, when we think individually, we attempt to respond—internally and vicariously—to the imagined responses of others to our ideas and arguments. Vygotsky's central claim was that to understand individual psychological development it is necessary to understand the system of social relations in which the individual lives and grows. This system is itself a product of generations of development over time, so that the individual is, in effect, historically situated, an heir to a long cultural development. Primary among the tools that, for Vygotsky, are each individual's patrimony is language, which mediates all thinking (cf. Wertsch, 1985).

Situated cognition and rationalism: incomplete accounts of human thought and learning

Conceptual rationalists and situated-learning theorists can each assemble convincing evidence in support of their views. Each appears, within its own terms, to offer a coherent account of human intellectual development. But each maintains its coherence by limiting the range of questions it is willing to address. Conceptual rationalists search for concepts that appear to be universal and focus their attention on the earliest emergence of these concepts. There is little attention to the varied forms that adult knowledge might take or to how more particular culturally mediated knowledge might develop out of biologically prepared structures.

Situationists, on the other hand, are interested in the ways that culture, history and immediate social contexts shape cognitive activity. They are very interested in different cultural systems of knowledge. They offer theories—such as Vygotsky's—of how participation in particular forms of social activity leads to personal cognitive competence. But these socio-historical theories of cognition have little or nothing to say about the contributions that the individual might make to development. They do not consider either the biological starting point of development—that is, the constraints that biological endowment might place on the directions of socially shaped cognitive development—or the complex ways in which individually constructed concepts might come into play in socially shared cognition.

On the whole, today's students of situated cognition are more interested in mapping details of how people co-ordinate cognitive activity in particular social and tool situations than they are in accounting for personal structures of knowledge. A problem within situated cognition theory is that the individual seems to disappear. Individual competence is replaced by social and institutional forms of behaviour. Individual knowledge and skill—characteristics of individuals that can be carried with them from one situation to another—are replaced by emergent cognition that belongs to no one and disappears when the moment of emergence has passed.

Situated rationalism: learning as tuning of prepared structures

The contrasts just drawn suggest that neither situated cognition nor conceptual rationalism can by themselves offer a very complete account of human learning, competence or performance. The two together, however, may be able to do what neither appears able to do alone: to provide an account of how individuals learn both the universal concepts for which they appear to be biologically prepared and the much greater variety of culturally specific knowledge and ways of acting that characterize mature people. My proposal for such a theoretical joining—which I call 'situated rationalism'—can be viewed as either re-introducing the individual into a radical theory of situated cognition or introducing the social into a theory of biological constraints on individual learning. The basic idea is, first, to expand the notion of prepared structures to include those that are socially prepared and, then, to suggest how the prepared structures brought into the situation by participating individuals might function to produce contextually specific learning and socially distributed cognition.

Conceptual rationalists argue that learning occurs when prepared structures are elaborated in the course of interaction with the environment. The prepared structures direct and constrain attention to particular environmental features that will support elaborations of particular concepts. They render the individual sensitive to particular affordances of situations. It is an easy extension to think of structures resulting from past engagements in culturally specific situations as similarly constraining the way in which individuals enter new situations. Once in a situation of engagement with the environment, prepared structures are modified and elaborated by that engagement. Engagement in a situation thus modifies the structures that prepare one for the next situation. It is this process of elaboration that we call learning.

The work of conceptual rationalists has focused on those elaborations that result in relatively permanent new structures, that is, those that will turn out to be tuned to the affordances of many future situations over an extended period of time. The situated-rationalist argument calls for just a slight shift in perspective, one that in no way challenges the underlying argument of learning on the basis of elaborating prepared structures. It suggests that, in each situation of engagement, what is actually elaborated is only what is needed to act successfully in that particular situation. The new conceptual elaborations are 'general' or permanent only to the extent that future situations afford their use. On any particular occasion, one attunes one's behaviour and, thus, one's knowledge to the demands of the occasion.

Learning, for the situated rationalist, is a matter of tuning to one's immediate situation, of becoming good in the situation in which one practices. The situation is inclusively defined. It refers to everything in the physical surroundings and the material used; to the social, institutional and personal purposes at play;

to the other people engaged; and to the language used. In short, much of what is traditionally viewed as context for learning is viewed in a theory of situated practice as an essential part of learning and, thus, of what is learned.

The heart of the argument, then, is that learning is a matter of passing through successive situations in which the individual becomes competent. Individuals develop this situated competence in each situation on the basis of their prepared structures. These prepared structures have both biological and socio-cultural roots, with the biological predominating in the earliest months and years and the socio-cultural taking increasing control thereafter as each individual's personal history of situations grows and initial biologically prepared structures are successively modified. (See Gardner, 1991, and Johnson, 1987, for convincing arguments that earlier, more purely biologically based schemas do not wholly disappear from adults.)

The learning processes are the same, whether the prepared structures are, in a given instance, primarily biological or primarily socio-cultural. In fact, because socialization into a culture begins at birth, there is probably no instance thereafter that can be categorized as purely biological or purely socio-cultural in its preparation. In each new situation, learning is a matter of beginning to act in the environment on the basis of the particular affordances of that environment. One's initial actions are either successful or not. If they are dramatically unsuccessful, that is, if there is no match at all between one's prepared structures and the affordances of the environment, the most likely response is to leave the environment, either physically, if possible, or by 'tuning out' when actual physical departure is not possible. If the match is complete, there is no need for learning. One just acts.

But if the match is partial—enough to keep one engaged, but not enough to provide a ready-made set of actions—a process of tuning to the affordances of the environment sets in. This tuning is what I mean by learning. It produces an ability to act 'perfectly' in the environment. But, because it is a tuning process, it results in a specifically situated competence. The competence developed will not be perfect for any other specific environment. An effort to specify the mechanisms of tuning would reach beyond what either situated cognition or conceptual rationalism has attempted to study until now. Connectionist models of cognition (Rumelhart, McClelland & the PDP Research Group, 1986), however, suggest a metaphor, at least, for what the process might be like. In connectionist models, a cognitive system learns by spreading activation across multiple nodes simultaneously. No single node embodies meaning or knowledge; rather, meaning is emergent, the result of on-going activity in the network. The state of the network (its nodes, the strength and directionality of its links) at the beginning of an episode interacts with new stimuli in the situation to produce a particular pattern of activity. This activity can be thought of as the tuning process. It produces a change in the network. The changed network will react somewhat differently—even to similar stimuli—at the start of the next episode and will again tune itself, in a more or less continuous cycle of situation-specific learning.

From skeletal structures to scientific concepts

I want to show now how the idea of situated rationalism plays out in two well-studied knowledge domains. I will not explore personal histories or describe details of the tuning process, because the necessary research—macro-longitudinal and micro-longitudinal combined—has not been much pursued until now. But it is possible, on the basis of an assembled body of research, to lay out some plausible hypotheses about the relations between biologically prepared and culturally elaborated knowledge structures in different domains.

There are, logically and empirically, at least two kinds of relations between biologically prepared and culturally elaborated structures of knowledge:

- The culturally accepted forms may be coherent with biologically prepared structures. For example, certain core concepts of number and algebra can be developed by elaboration of basic principles of counting, plus knowledge about physical material, that there is reason to believe are among the biologically prepared structures available to all human infants.
- Culturally accepted or scientific concepts may contradict beliefs that are rooted in biologically prepared structures. This appears to be the case for many concepts in physics, where the contradictions give rise to systematic 'misconceptions' and difficulties in learning scientific concepts. It may also be the case for certain mathematical concepts, such as fractions or proportions.

The nature of learning can be expected to be quite different for cultural concepts that are coherent with biologically prepared structures than for those that are contradictory. At the very least, we should expect to find differences between the two in simple ease of learning. Such differences would be reflected in the ages at which children in a given society acquire the cultural concepts of interest; in the ways in which knowledge of and mastery of the concepts are distributed in the population; and in the extent to which mastery of the concepts appears to depend on formal instruction. For the concepts used as examples here, all three of these indicators distinguish well between the coherent and the contradictory. Non-Newtonian forms of thinking about the physical world predominate among all but the best educated; formal physics is learned relatively late, primarily in formal institutions of instruction, and with considerable difficulty by most students. By contrast, core mathematical concepts that are derivable from knowledge about counting and physical material are learned easily, young and by nearly everyone who participates in any kind of market economy.

The examples I have mentioned from physics and mathematics are prototype cases, instances in which enough research has been done to make it clear whether we are dealing with a coherent or a contradictory relationship between the prepared and the culturally elaborated. In other instances, it is not clear what kind of transformation—whether an elaboration of a biologically prepared concept or its replacement by a new idea—must take place to reach a culturally accepted concept. For many of the most important learned concepts, a mixture of coherent and contradictory rela-

tions with several different prepared structures is likely. To explore the question of relations between culturally elaborated and prepared structures, however, it is useful to focus attention first on these clean, prototypical domains.

Additive composition of positive integers: origins of algebraic principles in biologically prepared structures

Much of elementary arithmetic has as its conceptual base the fact that all numbers are additive compositions of other numbers. This compositional character of numbers provides an intuitive basis for understanding fundamental properties of the number system. These properties include commutativity and associativity of addition, equivalence classes of addition pairs (additive composition), complementarity of addition and subtraction (additive inverse), and certain rules of distribution. Children appreciate these properties at a surprisingly young age, as shown primarily by studies of their invented arithmetic performances (see Resnick, 1986, for a summary of this research). Challenged to solve problems for which they have no ready algorithms, children invent procedures that can be shown to apply implicitly these principles. Similar reasoning takes place among minimally schooled adults carrying out arithmetic tasks as part of their daily work (e.g. Schliemann & Acioly, 1989). Together, these two lines of research point to a body of mathematical knowledge that appears to be easily and, probably, universally acquired. I show in the next section how a pair of algebraic principles might plausibly be elaborations of early biologically prepared structures.

Commutativity and associativity

Commutativity and associativity are distinct properties in number theory, but children appear to understand them as a single permission to combine numbers in any order. For example, here are the words of a child (7 years, 7 months) who considered this permission self-evident. He was asked to add 45 and 11 and then immediately afterward 11 and 45. He simply repeated his first answer and said, 'They're the same numbers, so they have to equal the same thing.' (Resnick, 1986, p. 166). A more sophisticated implicit application of commutativity and associativity was shown by Resnick and Omanson (1987) among second and third graders. Using a mixture of reaction time and interview data, they showed that several children added problems such as 23 + 8 by decomposing 23, yielding (20 + 3) + 8, and then reconfiguring the problem to (20 + 8) + 3. Because (20 + 8) could be recombined to 28 very quickly on the basis of place value knowledge, this allowed the children to apply a simple counting-on solution: 'twenty-eight ... twenty-nine, thirty, thirty-one.'

Two sets of skeletal structures seem to provide the biological foundations for learning commutativity and associativity. One of these, the structures underlying the rules for counting and numerical quantification, has been extensively

analyzed by Rochel Gelman and her colleagues (Gelman & Gallistel, 1978). The second, a proto-quantitative part/whole schema, is one of a set of structures for reasoning qualitatively about amounts of physical material that have been hypothesized by Resnick & Greeno (1992; Resnick, 1992). Although the necessary research on infants and very young children has not yet been done, we can plausibly hypothesize a skeletal structure that helps children develop understanding of how the physical material around them comes apart and recombines. Such a structure would specify that material amounts are additive. That is, one can cut a quantity into pieces that, taken together, equal the original quantity. Or, one can put two quantities together to make a bigger quantity and then join that bigger quantity with yet another in a form of hierarchical additivity. This proto-quantitative knowledge allows children to make judgments about the relations between parts and wholes. Children know, for example, that a whole cake is bigger than any of its pieces. They also know that the order in which sets of candies or sweets of different colours are poured into a bag does not change the total amount of available (Irwin, 1996). This latter knowledge about order is the basis for an early understanding of commutativity and associativity that can be expressed in the form of proto-quantitative equations:

(1) Part1 + Part2 = Part2 + Part1

(2) (Part1 + Part2) + Part3 = Part1 + (Part2 + Part3)

Knowledge of counting principles and of the proto-quantitative part/whole schema appears initially to interact very little in children's thinking. That is, young children will count in order to determine how many in a set, but they do not usually think spontaneously of using counting to solve problems involving compositions or decompositions of sets. It apparently requires some social provocation—informal teaching, if you will—to get children to combine their counting and their part/whole knowledge structures. When they do, however, a new prepared structure results, which can be labelled a quantified part/whole schema. This schema allows children to use counting and numbers to reason more precisely about amounts of physical material. They can now specify by how many a set is increased, for example, or exactly how much is left when a set of nine items is broken up into parts and three items are removed.

All of the relationships between wholes and parts that were present in the proto-quantitative schema are maintained in the new, quantified version. But now the relations apply to specific quantified amounts of material. As a result, children can now reason using quantified equations, such as:

(3) 4apples + 7apples = 7apples + 4apples

(4) (3apples + 5apples) + 4apples = 3apples + (5apples + 4apples)

Although numbers play a role in these quantified equations, they function at this stage as adjectives, that is, as terms that describe the properties of the quantities of material being physically or mentally manipulated. Eventually, however, numbers begin to take on a life of their own. They become objects in their own right, mathematical entities that can be reasoned about. When this happens, the same basic part/whole schema can organize knowledge about relations among numbers

themselves. The commutativity and associativity principles now apply not to physical material but to mentally constructed mathematical entities:

(5) $4 + 7 = 7 + 4$

(6) $(3 + 5) + 4 = 3 + (5 + 4)$

A final step in the elaboration of the part/whole schema is its extension to numbers in general, rather than specific numbers. At this point, children understand that commutativity and associativity are always true for addition, no matter what the numbers. Thus:

(7) $n + m = m + n$

(8) $(n + m) + p = n + (m + p)$

In this sequence, the relational structure of the original proto-quantitative equation (the 'skeletal structure') is maintained throughout, but the knowledge structure is elaborated by successive changes in the objects (first, unquantified stuff, then counted sets, then specific numbers, then numbers in general) that are related. The algebraic forms of reasoning and representation are socio-culturally transmitted tools of thought that enable and constrain individual elaborations.

The Newtonian laws of motion: cultural forms that contradict biologically prepared structures

In contrast to basic number concepts, certain scientific laws of physics seem to call on people to replace rather than elaborate their prepared structures for interpreting the motion of objects. A substantial body of research now documents the surprising difficulty that even well- educated people have in learning some of the basic Newtonian laws of motion (see, for example, Halloun & Hestenes, 1985; McCloskey, 1983; Viennot, 1979). During the early years of research on naive conceptions, organized alternative theories—often thought to parallel those of classical and medieval physicists—were attributed to students. In the past few years, the dominant view has been that students' spontaneous conceptions should not be characterized as systematic theories but rather as collections of ad hoc explanations. In the most extensive and radical reanalysis, diSessa (1993) has claimed that there is no single organizing principle or set of principles that gives rise to naive explanations of physics. Instead, these explanations are constructed out of a set of phenomenological primitives that are called on in response to specific situations. Some of diSessa's primitives are recognizable as the kinds of beliefs about objects and motion that students of infant cognition (e.g. Carey & Spelke, 1994) have been documenting for extremely young children. Presumably—although diSessa does not discuss their origin—they are rooted in skeletal structures that prepare infants to learn from interaction with the physical world. In this actual physical world, motions die away naturally, keeping an object in motion requires exertion of effort, and vertical and horizontal motions are experienced differently.

Another kind of analysis (Nersessian & Resnick, 1989) suggests that,

although particular explanations may be local and ad hoc, as diSessa and others claim, certain fundamental presuppositions underlie and constrain naive explanations of physics. People are not aware of having these commitments and do not normally articulate them, even under the kind of intensive probing that is often carried on in studies of physics conceptions. Indeed, precisely because they are presuppositional, taken for granted, they are powerful assimilators of new data and may create serious epistemological obstacles (Bachelard, 1980) to learning scientific concepts that require new ontologies (cf. Carey & Spelke, 1994).

I consider here two implicit presuppositions that could well be grounded in biologically prepared structures and that together can account for most of the evidence that has been collected on misconceptions about projectile and free-fall motion. Nersessian and Resnick (1989) have mapped findings in the research literature about naive conceptions of physics to an epistemological analysis of the historical shift in physics to Newtonian inertial theory. Our analysis identifies two presuppositions shared by pre-Newtonian scientists and today's naive physics thinkers. One, which we term the stasis presupposition, selects some situations as requiring explanations and others as natural and, therefore, not needing to be explained. The second, which can be termed the agency presupposition, sets criteria for an acceptable explanation.

The stasis presupposition specifies that being at rest is a natural state of objects. This means that an object at rest does not require an explanation. It just is. Motion, by contrast, is a change in state and, as such, requires an explanation. Our analysis suggests that typical naive conceptions of projectile and free-fall motion are all based on an implicit presupposition that motion requires an explanation. In their presupposition that motion requires an explanation, naive thinkers about physics are like pre-Newtonian scientists. Aristotle, Burridan, and even Galileo in the early part of his career, thought of motion as a process. It was distinct from rest, which they thought of as a state. Their ontology, like modern students', thus contained two separate categories: states and processes; rest and motion fell into different categories. In Newtonian inertial physics, however, uniform linear motion is incorporated into the state category. Change in motion (acceleration), but not motion itself, is considered a process. To construct a Newtonian explanation, then, requires overturning a fundamental, early belief, one that is plausibly rooted in bodily experience (cf. Johnson, 1987) of the difference between rest and exertion.

This basic physical experience would also give rise to the presupposition of agency. Naive physics thinkers appear to believe that explanations of physical events such as motion must include specification of a mechanism. They look for a causal agent that makes an event occur. This fits nicely with what we are learning about biologically prepared structures for causal reasoning. It contrasts sharply, however, with the structure of scientific explanations in physics since Newton. Scientific physics accepts as primary explanations expressions of mathematical relations among formally defined entities. The constraint equations that express physical laws do not specify agencies. Force, for example, increases just because

mass or acceleration increases and not because mass or acceleration is an agent of change.

The definition of motion as a change in state and the requirement that explanations include causal agents are not articulated by subjects; neither is the relationship between change and the need for explanation. These implicit definitions and presuppositions must be inferred from the general pattern of responses in preconceptions research. A good example of how preconceptions might derive from the stasis and agency presuppositions lies in the common belief that motion implies force.

Every study of naive conceptions of physics reports some variant of a belief that, if an object is in motion, there must be a force acting on the object. This belief fits naturally with the presupposition that motion requires explanation, and with the assumption that an explanation must specify a causal agent. If motion must be explained and if explanations require causal agents, a limited number of possibilities exist.

If one believes that there must be an agent of change when there is motion, one will probably look first for external agents: pushes, pulls, shoves, all kinds of direct mechanical actions of one body on another. The situations typically posed to students in naive physics studies, however, are ones in which an object is moving without any current external force acting on it. In these situations, the only possibility is that the agent must be inside the object. This leads to the frequently expressed conception that a force of some kind is stored inside the moving object. This idea is expressed by subjects in various terms, some borrowed from scientific language (energy, inertia, force, even potential force) and some borrowed from distinctly everyday language (oomph, power).

Consider now the possible biologically prepared structures that might give rise to these beliefs. The infant's responsiveness to information about external agents of motion has been documented by Carey and Spelke (1994) and others. Furthermore, the infant's experience of bodily expenditure of energy and observation of its relation to observed results in the world could plausibly be interpreted by skeletal structures that produce a concept of internal energy-producing motion. Thus, the documented naive physics conceptions could well be elaborations of biologically prepared structures. These would have to be replaced in order to adopt a Newtonian scientific perspective.

Situated rationalism and education

The distinction between concepts that are coherent with biologically prepared structures and those that contradict them suggests the need for two different approaches to teaching and instruction. For coherent concepts, 'teaching' is largely a process of helping children elaborate their initial, biologically guided concepts into particular cultural forms. It is more a matter of guided exposure to new opportunities for using concepts than of directly telling them about new ideas, although experience with a culture's conceptual language and tools must be

considered a key part of this exposure (cf. Resnick & Greeno, 1992). Most educators who espouse a 'constructivist' philosophy—a clear majority today—are, I believe, acting on an implicit assumption that what children already know as they enter an instructional situation is coherent with the new concepts to be learned. On this assumption, it is reasonable to conclude that the children's own cognitive constructions will move without much resistance toward the culturally accepted forms to which they are exposed.

In the case of cultural knowledge that contradicts biologically preferred concepts, however, education must follow a different path: still constructivist in the sense that simple telling will not work, but much less dependent on untutored discovery and exploration. For these contradictory concepts, ways of helping children replace rather than elaborate initial beliefs need to be found. We have not yet discovered very good ways of doing this. In physics education, for example, where the phenomenon of resistance to scientific concepts has been recognized for some time, an early idea was that confronting students empirically with the inadequacy of their initial concepts would stimulate rejection of the old and openness to new ideas. This has not worked very well. Students mostly find ways of reinterpreting the empirical data to fit their initial conceptions (Champagne, Klopfer, & Anderson, 1980; Johsua & Dupin, 1987). Even when they accept the inadequacy of their initial ideas, physical experience and data do not directly suggest new, scientific concepts. Some researchers have experimented with the use of analogies—usually embodied in special physical models—as a way of teaching the new concepts. Their reported successes (e.g. Brown & Clement, 1989; Sayeki, Ueno & Nagasaka, 1991; White, 1992) have always involved very large investments of time (for students and teachers alike) for learning very limited concepts. In another proposed approach (Chi, 1992; Chi & VanLehn, 1991; Ohlsson, 1992), students would be directly taught certain core scientific concepts and then guided through a process of applying those concepts to many cases. The idea is that students might at first apply the concepts rather mechanically but would eventually come to believe in them because they yielded intellectual power. At that point, but not earlier, it might be profitable to pit the new concepts against the initial, biologically rooted ones. This attractive idea still requires empirical testing.

Beyond the problems involved in teaching specific concepts, the situated rationalist view of learning and development suggests some new perspectives on traditional problems in education. In particular, individual differences and transfer, both central constructs in educational psychology, can be usefully reconstrued. Both of these constructs have been classically defined in terms of bundles of relatively stable skills. Over the decades, debate has focused on how to describe these skills and how they bundle together. The situated rationalist framework suggests that, instead of decontextualized skills, it would be helpful to think of personal histories as the important determinants of the way individuals will act in a particular situation.

When individuals move from situation to situation, they carry histories of prior experience with them. These are histories of ways of behaving. They include

the elaborated knowledge structures, along with affective and social propensities, developed in the course of tuning to prior situations. The way one enters a new situation is influenced by one's history of past situations. Situations experienced as similar to a past situation will initially evoke—not necessarily consciously—ways of behaving that developed through practice and tuning in the previous situation. If these ways of behaving work—that is, if they result in successful behaviour in the new situation—they will be further practiced and elaborated. On this analysis, 'transfer' would be a case in which a prepared structure and the new situation's affordances interact to produce a situated performance in accord with what educators have defined as the 'right' response. Lack of transfer would be a case in which affordances and prepared structures produce a tuned performance not in accord with educators' expectations.

Two features distinguish this notion of personal histories from traditional views of transfer of skills and knowledge. First, what one carries with one to a new situation is much more complex and organic than a collection of skills. It is a whole set of dispositions, interpretations and representations that, together and interactively, produce an initial response. Second, one's personal history is determinative only as one enters the new situation. Thereafter, all of the people, tools and material resources of the new situation shape a new situated practice. Cognition is emergent in the situation and specific to it. Yet the individual is not lost, for he or she leaves the encounter with a residue of preparation for the next situation.

The idea of personal histories as a way of thinking about individual differences and transfer suggests that we might profitably think of education as an effort to organize sequences of designed situations that are likely to prepare individuals to tune adaptively to the kinds of natural situations they will encounter outside designated institutions of learning. Current formal education does this very poorly (see Resnick, 1987). The special situation of the classroom—calling for private rather than socially shared work and isolating mental activity from engagement in the social and physical world—builds skills and knowledge that allow students to function in school but often fail to transfer to the worlds of work, civic and personal life. To change this thoughtfully and productively will require a form of theory that is now largely absent from psychological thinking and only loosely developed in other fields of social science: a theory of situations. Such a theory would define the dimensions—social, cognitive and physical—of situations with an eye to how activity in one situation might prepare individuals to enter another. Developing such a theory, taking into account both biological and social constraints on learning, represents a major challenge for those who would apply the concepts of situated rationalism to education.

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LEARNING TO THINK OR LEARNING TO MEMORIZE? A CONSTRUCTIVIST REFORMULATION OF AN OLD DILEMMA

Marcel Crahay

Do schools offer dead knowledge?

As long ago as 1929 A.N. Whitehead, in his work *The aims of education,* pointed out how important it is to avoid weighing children down with 'dead' knowledge. He wrote: 'In training a child to activity of thought, above all things we must beware of what I will call 'inert ideas'—that is to say, ideas that are merely received into the mind without being utilised, or tested, or thrown into fresh combinations' (p. 1-2). He was criticizing here the formalism of the most common teaching methods of the time, which equipped schoolchildren with knowledge that they were incapable of using to solve everyday problems.

Going back even further, in a book entitled *Talks to teachers on psychology* (1912), William James gave an example of 'dead' knowledge, drawn from classroom observation:

Marcel Crahay (Belgium)
Marcel Crahay succeeded Professor G. de Landsheere as professor of experimental pedagogy and educational psychology at the University of Liège. He also taught for three years at the University of Geneva. His publications include, in collaboration with A. Delhaxhe, a series of works entitled *Agir et interagir à l'école maternelle*, which are intended for teachers and take a 'Piagetian' approach to pre-school education. While Professor Crahay's current interests cover all aspects of compulsory schooling, his main aim continues to be to draw on the findings of genetic psychology so as to develop pedagogical methods that will benefit all children.

A friend of mine, visiting a school, was asked to examine a young class in geography. Glancing at the book, she said: 'Suppose you should dig a hole in the ground, hundreds of feet deep, how should you find it at the bottom,— warmer or colder than on top?' None of the class replying, the teacher said: 'I'm sure they know, but I think you don't ask the question quite rightly. Let me try.' So, taking the book, she asked: 'In what condition is the interior of the globe?' and received the immediate answer from half the class at once: 'The interior of the globe is in a condition of *igneous fusion*'.

More recently, Barnes (1976) and, subsequently, Bransford (1986) have taken up the concept of 'inert' ideas, maintaining that most of the knowledge that we acquire at school is alien to our own individual way of thinking. According to the former, 'School knowledge is the knowledge which someone else presents to us. We partly grasp it, enough to answer the teacher's questions, to do exercises, or to answer examination questions, but it remains someone else's knowledge, not ours' (p. 81). The same author goes on to point out that we need to distinguish between knowledge transmitted by the school and knowledge that the individual acquires through his own actions. The latter appears to be more solidly based and more firmly assimilated by children, who use it to make sense of the events that compose their daily lives.

Nowadays, the situation is still very much the same. Numerous studies on the didactics of mathematics and physics show that students can correctly answer questions that explicitly refer to the knowledge that they have been taught in school, but that they cannot solve problems that involve making use of such knowledge.[1] In the United States, wide-ranging studies of the effectiveness of school education indicate that, despite an improvement in basic skills, the higher cognitive processes are less easily learned (National Assessment of Educational Progress, 1981, quoted by Glaser, 1986). Such studies emphasize the school's inability to provide children with the kind of knowledge that can be used to solve problems.

Should we acquire facts or learn to think?

Such studies surely call for some adjustment to our educational systems. But how? The question needs to be answered carefully.

For some people the answer is obvious: we attach too much importance to the acquisition of knowledge, and should place more emphasis on the development of intellectual skills. That is the answer given in the document *Les socles de compétences dans l'enseignement fondamental et au premier degré de l'enseignement secondaire* [The foundations of skill acquisition in primary education and the first level of secondary education], distributed in 1994 by the Ministry of Education of the French Community of Belgium. It contains the following passage:

The purpose of education is to enable children to use what they learn at school outside the framework of their courses or subjects. In fact, however, young people's knowledge is very compartmentalized: they know a lot, but the things they know are isolated, fragmented

and separated from one another; [. . .] As a result, schools need to establish new priorities and different strategies: more than ever, they need to go beyond the mere transmission of knowledge and [. . .] give priority to teaching children how to live, how to think and how to do things, taking account of their age, their ability to think in abstract terms, the different ways in which they learn and the time necessary for the daily work of teaching. This approach requires the explicit employment [. . .] of pedagogical strategies that will encourage the progressive mastery by all children of transverse skills. These transverse skills form the basis of learning and of personal, intellectual and vocational development (p. 97).

Following on from this assessment, the report provides a long list of intellectual procedures presented with no reference to any content.

When such views are the result of collective thinking, they reflect a certain intellectual climate. In the world of education, the acquisition of knowledge is no longer fashionable. That being the case, there is a great temptation to summon up the old demons of formalism and revive the Manichaean opposition between transmitting knowledge and teaching people how to think. It is presumably this line of thought that has given rise to the techniques of cognitive educability, now in vogue in some pedagogical circles: skill enrichment programmes (SEP), logical reasoning workshops (LRW), Mialet's Cubes, etc., not to mention the devices of A. de la Garanderie (1982, 1984). These techniques postulate to varying degrees that it is possible to train pupils in the cognitive skills that form the basis of our mental processes. The content of the exercises proposed is largely irrelevant: what is important are the underlying processes: categorization, comparison, reasoning, etc. They can often seem rather abstract, with a strong logical connotation, like general intelligence tests. The reduction of actual content to the minimum is intended, on the one hand, to avoid reminding the pupil of previous failures and, on the other hand, to exercise cognitive abilities that can be applied to any complex task.

This idea is not new. The teaching of Latin has long been justified on the grounds that it provides a form of intellectual gymnastics that will develop children's mental abilities. And some people have made the same claim for mathematics or even information technology. Indeed, has not Montaigne's precept 'Better a thinking mind than a head stuffed with facts' become part of popular wisdom among his compatriots?

In *L'évolution pédagogique en France* [The development of educational methods in France], Durkheim (1990) offers a well-judged critique of this famous aphorism. In particular, he shows the extent to which Montaigne doubted the capacity of pedagogy to form judgement, which he believed to be innate, in those fortunate enough to be endowed with it. Thus, Durkheim points out that:

There are many well-known passages in which Montaigne compares the mind to a vase into which knowledge is poured. Just as the shape of a vase does not depend on the liquid that it contains, so the form of the mind is independent of the knowledge within it. Knowledge cannot alter its form. Knowledge does not produce soundness of judgement, and sound judgement can exist without any kind of knowledge (1990, p. 257).

Montaigne's *Essays* can hardly be regarded as a best-seller, and his works are rarely read by teachers in their original form. If his remarks are often quoted it is because they offer an apposite formulation of widely held views. One such view is that it is possible to define intellectual processes independently of their content. In fact the content is seen as a pretext, object or simply opportunity for developing thought processes.

What we should do is to encourage educationists to go back to reading Durkheim who, at the beginning of this century, asked questions which are still relevant today and which provided answers that foreshadow the lessons of contemporary psychology.

> So, despite our former rejection of it, are we not coming back to formalist pedagogy? The general aptitude to think, judge and reason is, it seems, a collection of completely formal aptitudes that are independent of any specific subject-matter. So far, we have not even stated that secondary schools should teach this rather than that, this particular area of positive knowledge rather than any other. Could it not be that the nature of such knowledge and its importance are in our eyes secondary, indeed more or less immaterial, and that our pedagogical ideal will therefore be strangely similar to the ideal pursued by the schools of scholastic philosophy or the colleges of humanists? Might not our objective be to train the mind in a general way rather to furnish it and feed it?
>
> Not at all, since it is impossible to teach someone to think unless there is a specific object to think about. We do not think in a vacuum. The mind is not an empty vessel that can be shaped in just the same way we shape a glass and then fill it up afterwards. The mind is made to think about things and it is by making it think about things that it is formed (1990, p. 364).

Here we have the key to this article. In our opinion, intellectual processes cannot be defined independently of the knowledge or content to be mastered or the problems to be solved. More specifically still, there is no point in defining intellectual processes or aptitudes which the pupils should have at their command, without specifying the area of knowledge or situations in which this process should be applied. Throughout this article we shall endeavour to follow in Durkheim's footsteps and endorse his views with arguments based on contemporary psychological research and, more specifically, on Piaget's constructivist theory. In other words, we shall endeavour to show that cognition cannot develop independently of the contents to be handled or the knowledge to be built up.

The coaching of general thought processes: a rationalist aberration!

Over the last ten to fifteen years, teaching programmes and school textbooks have been devised in the United States of America with a view to encouraging pupils to think, solve problems and develop the capacity to learn. Glaser (1986, p. 255-59) distinguishes four kinds of curricula:

— Programmes orientated towards processes (Whimbey & Lockhead, 1980; Feuerstein, et al., 1980).
— Programmes which use familiar knowledge (Covington et al., 1974; De Bono, 1985).
— Programmes which focus on the teaching of heuristic principles for problem-solving in clearly structured areas (Rubinstein, 1979; Wickelgren, 1974).
— Programmes which are intended to stimulate logical thinking during the acquisition of basic skills (Lipman, Sharp & Oscanyan, 1979).

According to Glaser:

> In nearly all cases, these programmes are directed towards the teaching of general processes—heuristic principles, methods of reasoning and of solving transferable problems—which can be acquired. As for the contents, they involve either abstract tasks, such as the solving of 'puzzles', or situations from real life. These programmes seek to avoid the complex contents of most branches of learning. Indeed, the general view is that the difficulties involved in understanding these contents would take up too much of the resources needed to practice using and to acquire other processes of thought (1986, p. 259).

He concludes: 'Underlying all this, undoubtedly, is a question of theory and of knowledge of human thinking' (ibid.).

In fact, it is easy to identify in such educational practices the mark of the first cognitivist theories, which occupy an enormous place in the first generation of studies on artificial intelligence (AI). Andler, who makes it clear that he does not share this approach, explains:

> The way in which cognitivism initially conceived of cognition may be stated as follows: the cognitive system [. . .] is (or rather includes) an innate formal system which works on representations: the latter are expressions of an internal formal language, and the system, guided by formal rules, subjects them to calculable transformations, in which the form of the expressions and of the rules is the only operative factor (1987, p. 8).

Further on (p. 11), the author presents AI in the form of an equation: AI = *inference + testing + knowledge.*

In this paradigm, inference and testing play an antagonistic but complementary role. Inference enables an initial cognitive state to be transformed into another state. As for testing, it 'plays a decisive role in preventing an explosion of different combinations [. . .]'. In theory, any problem can be solved through the application of a proper algorithm of operations. Unfortunately, since several different possibilities exist at each stage and since rather a large number of stages has to be gone through in order to reach a solution, the number of operations that the computer (or the brain) must examine becomes much too great for the solution to be found by such means in a reasonable amount of time. In a game of chess, the number of possible series of moves has been estimated at 10120, which inspired Andler (1987, p. 12) to observe, humorously, that 'if the largest, fastest-

working computer had set out to decide on its first move at the time of the big bang, it would still be a long way today from having reached a conclusion'.[2] What is needed is therefore a heuristic approach, although heuristics, unlike an algorithm, does not always lead to the right, or the best solution.

According to Lindsay and Norman:

The strategies used are more like empirical rules; they are tactics for seeking relatively easily applicable solutions and are often only chosen on the basis of their effectiveness in solving problems encountered previously. Some of the most useful heuristics involve 'breaking down a problem into subproblems' and 'seeking analogies' (1980, p. 544).

Andler has something to say on this point too:

The need for a plentiful supply of highly structured knowledge has only gradually become apparent. At the beginning AI hoped to bring intelligence out of the use of general problem-solving procedures. About 15 years later, however, it reached the conclusion that most tasks, even those apparently requiring erudition, require a considerable amount of specific information (1987, p. 12).

Despite this, classical cognitivist models, struggling with the question of the most economic representation of knowledge, continue to see a base of knowledge as independent of inference and of testing. It is hard not to be reminded by this division of the rationalism of previous centuries and not to call to mind that old, though still relevant remark by Paulus: 'A closer look obliges us to agree with Spearman's view that although the faculties approach may lose all the battles, it always wins the war' (Paulus, 1965, p. 116).

However, it has to be recognized that the latest developments in cognitivism and AI research would go beyond these theses. Thus, the most recent research on problem-solving among experts and beginners has revealed the influence of structures of knowledge linked to a specific field, thereby repudiating previous studies which emphasized the role of 'pure' and supposedly transferable problem-solving techniques (Newell and Simon, 1972). In this connection, Minsky and Papert (1974) examined the transition from an approach centred on a general capacity for intelligent thinking to an approach centred on the basic knowledge that the individual possesses. According to these authors:

the knowledge-based approach assumes that progress is achieved through a better way of expressing, recognizing and employing varied and specialized forms of knowledge [. . .] In fact, it is not all obvious that intelligent people display a higher level on their general thought processes. Their intelligence may simply be a more profound understanding of organizing knowledge (quoted by Glaser, 1986, p. 264).

Encouraged by this, Chi, Lesgold and Glaser (1985) carried out a series of studies whose aim was to understand how experts solve problems that they encounter in their particular field of study (physics, in particular mechanics and radiology) and how their expertise is built up. Their performances were analysed around the

focus of one central question: To what extent are the reasoning processes of experts, as opposed to those of novices, determined by the way they organize their basic knowledge? Empirical studies show that the beginner's knowledge is organized around specific objects, explicitly mentioned in the enunciation of the problem, whereas the specialist's knowledge is organized in terms of principles and abstractions subsumed by those objects. These principles do not appear in the enunciation of the problem, but are mobilized by the experts on the basis of their knowledge of the particular field of study.

The difficulty that beginners experience in solving problems may thus be primarily attributed to inadequate basic knowledge rather than to limited intellectual capacities, e.g. inability to use heuristic strategies.

Recht and Leslie (1988) have also provided solid proof of the crucial role played by specific knowledge in the process of understanding an item of information. These researchers chose a group of good readers and a group of bad readers at the same school level. They then gave them a test on their knowledge of baseball so as to identify within each of the two groups those who knew quite a lot about baseball and those who knew very little. They then asked each student to read a text of 625 words about a baseball game. Their level of understanding was then assessed on the basis of three criteria: first of all, subjects had to reproduce with the aid of figurines the main phases of the game and comment on them; they then had to summarize the text and, finally, recall the twenty-two most important phrases. The results of the experiment make it quite clear that a poor reader who knows a lot about baseball is more likely to perform the tasks described above successfully than a good reader who has no idea about the game. As Tardif (1992) has pointed out, 'It is the pupil's specific knowledge that will determine whether or not he can deal significantly with the information presented, and this is equally true for reading, writing, mathematics and science' (p. 231). On the basis of these psychological facts Glaser has constructed a strong pedagogical hypothesis:

> The ability to solve problems, to understand and to learn has its roots in knowledge: the individual will always try to understand and analyse new data on the basis of what he already knows. It thus seems preferable to exercise learners' ability to solve problems or to correct mistakes of understanding in fields of knowledge that are familiar to them. The ability to make inferences and to generate new information may be stimulated by first of all encouraging pupils to draw upon their previous knowledge and then leading them gradually to restructure and enlarge that knowledge (1986, p. 270-71).

PREVARICATION BY PIAGETIAN PEDAGOGUES

Certain critics will feel obliged to point out that the cognitivists had to go a very long way round to rediscover what the Piagetians had been saying for years. The writings of Duckworth (1972) and, subsequently, of Kamii and Devries already warn against dissociating process from content.

> The desire to emphasize processes or 'cognitive knowledge' is a reflection—according to these two authors—of mechanistic and/or empiricist postulates which view intelligence as an information processing machine. The pedagogical application of such a view resembles an attempt to add gears to the machine and to adjust the mechanism. Education can also be seen as an unending series of improvements to a computer program enabling it to process an infinite amount of information. One of our past mistakes as empiricists was to believe that once a child had acquired a logico-mathematical structure, nothing else was needed except to apply this logical mathematical machine to all sorts of objects. In fact, [. . .] the structure does not exist independently of the content (Kamii and Devries, p. 24).[3]

It is interesting to recall that these lines were written at the height of the compensatory education movement. At that time, a number of English-speaking researchers were involved in the planning of pre-school programmes in an attempt to combat the socio-cultural handicaps said to afflict children of ethnic minorities. Some teams such as Biber, Shapiro and Wickens (1971) used Piaget's theory as a basis for teaching programmes whose main aim was the development of methods enabling children to learn to learn. These researchers obviously have something in common with those, such as Feuerstein, et al. (1980) or Lipman, Sharp and Oscanyan (1979), who advocate techniques of cognitive educability. On the other hand, the position taken by Kamii and Devries (1978) lies at the opposite extreme. But both claim to be following Piaget. Perhaps some clarification is in order.

In Piaget's work, it is the *concept of the overall structure* which appears to cause the greatest controversy. Those who still accept this concept believe that a subject who has displayed a mode of reasoning belonging to the operatory stage in a particular situation has mastered the corresponding cognitive structure and, consequently, he or she is assumed to be able to reproduce this form of reasoning in other situations. In the same way, if a subject has used procedures belonging to the pre-operatory level, we would be inclined to describe the subject as pre-operatory and to consider that he or she is essentially using thought processes characteristic of that level. To question the relevance of this inference would be to question the validity, usefulness and necessity of the concept of the overall structure.

In 1977, Vergnaud drew attention to the dangers of over-hasty efforts to define formal structures. He wrote:

> In defining so many different operations solely in terms of algebraic structures or ad hoc structures invented by him and then elaborated by logicians, Piaget acted hastily and imprudently: in pursuing his 'structuralist' aims, he ignored a large number of different forms of behaviour. If we examine children's behaviour content by content and task by task, we are led to doubt the usefulness of his description of children's thought processes at various stages in terms of general abstract structures' (p. 110).

In short, according to Vergnaud, it is too early to try to describe development in terms of a succession of overall structures.

Similar remarks are made by Flavell (1982), for whom the definition of the structures that characterize the stage of concrete operations and the stage of formal operations is not sufficiently clear and coherent. Their meaning is not absolutely clear and they do not appear to provide an accurate and complete description of different ways of thinking. Flavell also points out that the concept of 'stage' implies a certain uniformity of the forms of knowledge in a particular age-group, and empirical studies are far from confirming this hypothesis. The studies by Rieben, de Ribaupierre and Lautrey (1983) highlight the wide differences between individual subjects who apparently belong to the concrete operatory stage. This observation leads the authors to remark that 'if there is heterogeneity, it could mean that different forms of cognitive development exist' (p. 178).[4] Much the same conclusion is reached by J. Bideaud (1988), whose review of the studies on mastery of inclusion and transitivity compelled him to recognize that these two pillars of logical thinking are not constructed by children once and for all. We must, therefore, accept the evidence as it stands: if we examine the logical arguments and the empirical data in favour of the thesis and those arguments and data which invalidate it, we are led to the conclusion that cognitive development is far from being a fixed sequence of general stages.

The construction of a hierarchical system of different forms of knowledge within a particular sector finds its main justification on the theoretical level. The practical benefit of such an approach is, however, more obvious: an analysis of the links between forms of knowledge may directly influence the organization of learning. However, even within a particular area of knowledge, the development of children's abilities can seldom be described in terms of a single sequence (Crahay and Delhaxhe, 1983). Increasingly, it seems reasonable to abandon the unilinear view of the development of knowledge and to replace it by a multilinear, contextual approach.

Neo-Piagetian theories have clearly moved in this direction. Thus, for Fischer (1980), although development must be seen in terms of the elaboration and increasing complexity of cognitive capacities, these different capacities are most often generated independently of one another. More specifically, Fischer believes that, in all fields of knowledge, development proceeds in levels.

> Development is relatively continuous and gradual, and the person is never at the same level for all skills. The development of skills must be induced by the environment, and only the skills induced most consistently will typically be at the highest level that the individual is capable of. Unevenness in development is therefore the rule, not the exception (p. 480).

In addition, Riegel (1976), quoted by Lefebvre-Pinard (1980, p. 61), has shown that adults do not necessarily always use the most advanced level of cognitive operations at their disposal, but tend rather to show flexibility in adjusting their level of cognitive response to the nature and complexity of the situations or problems they encounter. In short, all these neo-Piagetian writers have seen fit to abandon the original concept of *stage* in favour of a multilinear and contextual view of development.

This in no way contradicts the basic thesis of constructivism, but compels us—in our opinion—to describe the general cognitive development of individuals in terms of vectors of development rather than in terms of a succession of structural states. This inevitably emphasizes certain features that are present in Piaget's writings. It was Piaget, in fact, who pointed out that the development of knowledge takes the form of 'decentration' (that is to say, a gradual distancing from one's own perspective), increasing mobility and an enlargement and increasing complexity of interactions with the environment.[5] It is surely not insignificant that in one of his last works (Piaget, 1975),[6] Piaget himself talks of a 'complete revision' and replaces the term 'equilibrium' by that of 'equilibration', so as to indicate a process and not just a succession of static forms of equilibrium. This new formulation represents a much broader approach. In particular, the notion of increasing 'equilibration' enables us at the epistemological level to more fully integrate the always incomplete nature of knowledge and, at the psychological level, to go beyond the idea that the stage of formal operations represents the final phase of development.

Piaget's thought has never stopped evolving. This has led Montangero (1995) to distinguish four phases in Piaget's work.[7] In its final version, the epistemological status of his theory takes on a much sharper focus, since it has become clear that constructivism is more than a hierarchical systemization of knowledge, but is above all a theory of how the child, in seeking to maximize his interactions with his environment, constructs himself by integrating both the results and the mechanisms of his thinking. This theory of the individual as the creator of knowledge should lead us to a pedagogical model where the acquisition of knowledge is the fruit of a creative act, in which the child, confronted by problems of adapting to his environment, is engaged in a process of actively constructing his own way of thinking.

This attempt to redirect the focus on to the epistemological aspect means that the acquisition of knowledge cannot be examined in isolation from the mobilization of thought processes. More specifically, this assumes that, by encouraging among each of his pupils a process of active thinking, the teacher encourages the emergence of increasingly useful forms of knowledge. To put it yet another way, two pedagogical implications can be drawn from this neo-Piagetian perspective and can be expressed in the form of the following principles.

Principle 1. Educational activity should not seek to stimulate a process in isolation from a content to be structured. Consequently, the planning of any educational activity must begin with the definition of content, an aspect of the real world, or a situation.

Principle 2. The content or the situation is more than a context that is conducive to the stimulation of the process. The process only makes sense if it leads to the creation of new knowledge that is increasingly closely adapted to the real world. It is only on this condition that the process may be described as constructive.

These two principles, drawn from Piagetian constructivism, need to be reconciled with the pedagogical approach formulated by Glaser (1986), which derives from

research into cognitive psychology. However, for followers of both contemporary cognitivism and Piagetian constructivism, questions inevitably arise about teaching methods. More specifically, we need to try to identify which practices are likely to help pupils to construct dynamic knowledge that is applicable to real situations/problems.

How to help pupils to construct usable knowledge

LEARNING THROUGH SITUATIONS/PROBLEMS TO CONSTRUCT USABLE KNOWLEDGE

It seems to us that many teachers are prisoners of a 'top-down' conception of the relationship between knowledge and the application of this knowledge to a situation or problem. Many teachers believe that, as soon as the pupil has fully understood the theory, he only needs to apply it so as to solve the problems put to him. This conception is also to be found in the specialist literature. Thus, in 1974, Gagné suggested that teachers should plan their teaching in eight stages: (1) motivation; (2) initial contact; (3) acquisition; (4) retention; (5) recall; (6) generalization; (7) performance; (8) feedback.

Clearly, this pedagogical model separates the acquisition of knowledge from its exploitation and places these two facets of learning in a sequential relationship. The system of classification suggested by Bloom et al. (1956) also reflects this *top-down* approach. In particular, it assumes that many problems can be solved through the *application* to particular cases of knowledge that has been acquired and learned at an earlier stage. Similarly, the *knowledge application* linkage implicitly contains the idea that the most general rules or principles must first be mastered before they can be used in specific contexts. We have now been forced to recognize, however, that knowledge of principles, rules and laws does not guarantee that they will be properly used.

Studies carried out over the last decade on arithmetical problems are illuminating here. It would seem reasonable to believe that once children master addition and subtraction, they are able to solve all the problems that involve these operations. This is not so. We now know that the difficulty of arithmetical problems is only very partially explained by the operations that need to be carried out (addition, subtraction, multiplication and division) and by the size of the figures involved. Among the more important factors that affect the ability of young children to solve a problem are: the *semantics* of problems, the *position of the unknown factor* and the *formulation* of the statement (Fayol, 1989). Riley, Greeno and Heller (1983) identify four major semantic groups of problems:

1. Problems of change ('X had three marbles and Y gives him five. How many marbles does X have now?' or 'X had five marbles and Y takes three away from him. How many marbles does X have now?'). These problems always involve a transformation as a result of addition or subtraction.

2. Problems of combination ('*X* has three marbles and *Y* has five. How many marbles do they have together?'). These problems assume the combination of two sets without any transformation.
3. Problems of comparison ('*X* has three marbles and *Y* has five. How many fewer marbles does *X* have than *Y*?' or 'How many more marbles does *Y* have than *X*?'). These problems assume the ability to establish the numerical difference between two separate quantities.
4. Problems of equalization ('*X* has three marbles and *Y* has five. How many more marbles does *X* need to have the same number as *Y*?' or 'What must *Y* do to have the same number as *X*?'). These problems assume the ability to both make a comparison and effect a transformation. In the above examples, situations/problems that differ in their content are solved by using an identical arithmetical procedure. Thus, subtraction is involved in problems of change, comparison and equalization. However, children find it easier to handle the problems involving change than problems involving comparison and equalization, and that remains true up to an advanced level of schooling (third year of primary school). In other words, children are capable of using subtraction in some situations but not in others.

The position of the unknown factor also affects the pupils' performance. In problems involving change ($x + y = n$ or $x - y = n$), the unknown factor may be the initial quantity (x = before the transformation), the quantity that is added or taken away (y) or the final quantity (n = after the transformation). In the transformations resulting from combination ($x + y = n$), it may be the final quantity (n) or one of the subsets (x or y). In general terms, finding the *final state* in problems involving change and combination hardly presents any difficulties for children, even at nursery school. On the other hand, identifying the *initial state* (for example, '*X* had some marbles. *Y* gives him five. *X* now has eight. How many did he start with?') or one of the subsets in problems of combination (for example, '*X* and *Y* have eight marbles together. X has three of them. How many does *Y* have?') presents enormous difficulties (Fayol, 1989).

Finally, studies carried out by Hudson (1983) show how the formulation of the problem affects children's performance. This American researcher studied problems of comparison which assume the ability to establish the numerical difference between two quantities. In this author's view, the difficulty that children have in solving these problems may be due to failure to understand the way the problem is generally formulated rather than an inability to establish the numerical difference between the two quantities. The traditional formulation of this kind of problem contains comparative terms (more than, less than) which the young child does not understand. In order to test his hypothesis, Hudson compared the performances of children when the problem of comparison is presented in the most usual formulation ('How many more butterflies than flowers are there?') with their performance when the statement of the problem does not contain the comparative term ('How many butterflies will not have flowers?'). The results show that there is a clear difference in the success rate for these two formulations. From

the age of five onwards, most children solve the problem correctly when it is put to them in the 'How many will not have?' form. This experiment has also been carried out in Belgium with the same results (Delhaxhe & Godenir, 1990).

All these studies show that the ability to solve mathematical problems is not only a matter of applying formulas or arithmetical operations that have been learned beforehand. The pupil must, above all, be able to decode the situation/problem: he will utilize a particular procedure on the basis of what strikes him as significant in the way the problem is set out. Many writers now believe the representation of the problem to be the phase that has the most critical impact on the individual's performance (Andre, 1986; Best, 1986; Gagné, 1985; Glover, Ronning & Bruning, 1990; Newell & Simon, 1972; Voss, 1989). Thus, for Gagné (1985),[8] this stage is decisive since it is the way the problem is represented that determines what particular skills the subject decides to activate in his long-term memory in the search for solutions. Fayol (1989) and Resnick and Ford (1981) are even more specific: the child represents the problem to himself by interpreting the statement on the basis of his previous knowledge. Thus, Fayol (1989) has observed that, when confronted with the arithmetical problems described above, young children tend to simulate the actions described in the statement of the problem. They reproduce the situation as it develops in actions. When the problem can be stated in such a way that it can be easily expressed in exteriorized or interiorized acts, then it can be solved without difficulty. On the other hand, when the child cannot imagine the transactions of the problem, he will not apply the correct procedure for solving it, even if he is capable of using it in other situations. As for Resnick and Ford (1981), they point out that:

> The first step in any problem-solving situation is to build a representation of the problem; that is, to notice features of the problem and encode them in such a way that they are interpretable by the information-processing system. The information given in the problem statement must be encoded in forms that match elements in the individual's knowledge structure. This is what allows one's knowledge to be applied to the problem at hand (p.214).

Brown, Collins and Duguid (1989) believe that teachers should radically re-examine the relations between knowing how to do something in theory and knowing how to do it in practice. According to these authors:

> The breach between learning and use, which is captured by the folk categories 'know what' and 'know how', may well be a product of the structure and practices of our education system. Many methods of didactic education assume a separation between knowing and doing, treating knowledge as an integral, self-sufficient substance, theoretically independent of the situations in which it is learned and used (p. 32).

According to these same authors, we should adopt a completely different position according to which all knowledge is contextualized; they write:

> On the contrary, we maintain that the situation and the activity through which knowledge

develops are not elements parallel to learning and understanding: they form a basic part of them. One could even say today that learning and understanding are profoundly contextualized.

Much the same point of view is put forward by Resnick and Klopfer (1989), who believe that teaching will be that much more effective if it operates within a context of tasks which actually exist in the world outside the classroom. Likewise, Glaser (1986) writes: 'Effective processes of thinking would appear to derive from knowledge acquired in context, that is to say, from knowledge which is not dissociated from the conditions and constraints affecting its application' (p. 268). In short, we need to encourage teaching through problem-solving since it enables knowledge to be acquired in the context in which it is later to be used.

It would be wrong to claim that these conceptions are totally new. They can be traced back to Dewey, who criticized the formalism of classroom teaching and doubted the utility of knowledge acquired in the form of concepts. For him, the decontextualized teaching of concepts and principles could only be justified on condition that: 'there will be transfer of training from the well-ordered and well-labeled subjects to complex problem situations in which subject matter, as such, loses its identity' (in Beauchamp, 1957, p. 18).

SHOULD PRIORITY BE GIVEN TO TEACHING PROCEDURES?

Recent research in cognitive psychology has thrown new light on this important question.[9] It is not so much the distinction between *declarative knowledge* and *procedural knowledge* which we find interesting, as the way in which contemporary cognitivists understand the relations between them.

In his well-known work *The architecture of cognition* (1983), J. Anderson took up the distinction introduced by the philosopher G. Ryle, who, in 1949, distinguished between two forms of knowledge: *knowing that* and *knowing how*. It was also in that work that the American psychologist showed that it took only a few minutes to memorize an item of declarative knowledge, whereas learning a new procedure is a gradual process that often requires considerable effort. In Anderson's view, to convert one form of knowledge to the other is not easy (1983). The *proceduralization of declarative knowledge* has recently been much studied: through the regular application of our declarative knowledge we move from the slow and conscious application of rules to increasingly automatic and unconscious action. This process cannot be reduced to the mere application of general rules in particular contexts; there is a genuine transformation from the status of declarative knowledge into that of procedural knowledge. In other words, there is a quasi-ontological difference between the two forms of knowledge. While the former deals with the properties of things, the latter is closer to plans of action or rules of production stored in the form 'condition—action' (if *A* occurs, then so does *B*). The acquisition of procedural knowledge thus involves establishing sequences of conditional actions, which allow certain tasks to be

carried out in certain conditions. To read and understand a text, write a composition, spell, calculate, solve a problem and master a foreign language are all abilities that depend on procedural knowledge.

By highlighting the crucial role played by procedures in cognitive functioning, modern psychologists are encouraging a pedagogical revolution. As Coquin-Viennot and Gaonach (1995) have pointed out, the notion of knowledge as an automatic reflex is not terribly popular with teachers, who pride themselves on enabling children to discover laws or concepts for themselves. For many of them, once something is understood it has been learned. This is probably true of declarative knowledge, but not of procedural knowledge, which requires exercise or, better still, automatization. However, if, as is suggested by Brown, Collins and Duguid (1989), and Resnick and Klopfer (1989), and earlier by Dewey (1963), we show a preference for learning in practical situations, we run the risk of favouring procedural knowledge to the detriment of declarative knowledge. It is not difficult to identify the question underlying these various lines of thinking: if, accepting the view of contemporary cognitive psychology, we distinguish controlled processes which rest on the interpretation of declarative knowledge from the automatic processes which assume the utilization of procedural knowledge, we need to examine what place should be allocated to each of them. In order to answer this important question we must be fully aware of what is at stake and able clearly to define the advantages and drawbacks of each mode of interaction with our environment.

It is now generally accepted that we need to distinguish between two kinds of memory: *long-term memory* (LTM) and *short-term memory* or, rather, *working memory*. While the former is a means of storing what has already been learned, the latter acts as a means of processing all new information from the environment. More specifically still, it is through the working memory that external *inputs* are assimilated. It is also the working memory that integrates external *inputs* into the structure formed by previous knowledge. It is also the working memory which recovers knowledge stored in the LTM so as to generate the desired responses. To sum up, it is the working memory which processes the information filtered through the sensory receptors and the information stored in the long-term memory in the light of the requirements of the task to be performed or, more specifically, in the light of the way the subject represents that task to himself. It is also from this centre that the orders are sent out to activate the generator of responses.

This 'processing centre' operates under two important limitations: the length of time that information stored in the working memory is accessible and the amount of information that it can handle at any one time. According to Murdock (1961), and Peterson and Peterson (1959), items of information are only accessible for about ten seconds and then disappear from the field of consciousness if the subject does not recall them in one way or another. In other words, unless the subject is constantly accessing an item of information, it will disappear from the working memory. In addition, in a now-classic research study, Miller (1956) showed that the working memory can only hold 7 (± 2) items of information. This being the case, it seems clear that it can easily become blocked.

How is information overloading to be avoided in the working memory? Research has provided two answers to this question:

1. *The working memory may process items of information of widely differing dimensions.* Thus, in teaching children to read, a 'processing centre' may be occupied by a letter, a syllable, a word, a sentence or even a paragraph. In other contexts it may be a concept or a group of concepts. In other words, the working memory processes the items of information in the structured form in which they are stored in the LTM. In such circumstances, it is immediately apparent that the way the knowledge is organized is of great importance: the more highly structured the items of knowledge are the less space they will occupy in the working memory.
2. *The LTM may directly control the generator of responses.* However, this occurs if, and only if, the knowledge has become totally automatic. Otherwise the information stored in the LTM must pass through the working memory. The automatization of procedures has important advantages for the cognitive functioning of a learner and of any individual who is seeking to solve a problem: the more automatic processes an individual has at his command, the more he is able to activate responses without overloading the working memory.

The organization of knowledge and the automatization of processes are now thought to be the two main distinguishing features of cognitive skills. The superiority of automatic processes over controlled processes is clearly shown by the following table taken from Gaonach and Passerault (1995, p. 61):

TABLE 1. Comparison of controlled and automatic processes

Controlled processes	Automatic processes
are slow	are rapid
difficult: they require an effort on the part of the subject	simple
inhibitive of other processes: their activation hinders the operation of other processes; thus, no parallel operation is possible.	not inhibitive of other processes: they operate independently of the limitations of cognitive resources; thus, parallel operation is possible.
it is possible to exercise control over the operation of these processes, i.e., to deliberately prevent them from occurring.	these processes cannot be restrained: they cannot be stopped once the external conditions that set them in motion are met.

However, this synoptic table clearly shows the drawback in automatic processes: they are carried out almost unconsciously, and errors may occur fairly frequently. The research into the learning and use of algorithms of calculation is illuminating in this context. It indicates that the systematic errors that occur are mainly due to a kind of blind operation of the automatic processes: it is as if the children were trying to solve problems without exercising any semantic control

whatsoever over the processes or the results to which they lead. It is against this background that we should understand the queries raised by Fayol (1989) on the place of the training of elementary skills in building up basic mathematical ability. He writes that:

> It is obvious that an increase in the speed and accuracy of replies—the objective of this kind of training—may be particularly important for some kinds of activities, in particular because our processing capacity is limited and thus the more attention is devoted to the more elementary procedures, the less is available for handling more advanced activities. The main question is not whether or not training for automatic processing is necessary—the answer is obviously 'yes'—but how to choose the best method, enabling the subject to acquire accurate and rapidly accessible knowledge. The fact is that no one has so far managed to solve this problem in a completely satisfactory way (p. 197).

In the same work, the writer suggests a possible line of approach:

> Whenever we attempt to 'set up' a procedure that does not allow the subject any semantic control over the operations that he is carrying out, we run the risk of leading him into systematic errors and, worse still, strengthening erroneous associative links. It thus seems essential [. . .] that learning should take place in a meaningful context, even if subsequently training is directed towards the acquisition of automatic responses (p. 146).

How can one encourage significant learning?

Piaget's answer to this question is well-known. In his view, an object only has significance in relation to the schemas of assimilation that the subject applies to it. Thus, in his work *Les liaisons analytiques et synthétiques dans les comportements du sujet* (1957*b*), he writes: 'Broadly speaking, the significance that an object has for a subject in a particular situation is the union or intersection or structure of the schemas of actions that are applied to that object in that situation' (p. 50). This definition is supplemented in the following genetic epistemology study (*La lecture de l'expérience,* 1958); 'when an object is converted into a schema it acquires one or more meanings and it is this attribution of meanings which involves—even when it is the result of observation—a relatively complex system of inferences' (p. 59). In fact, this idea had already been developed in 1957 in *Epistémologie génétique et recherche psychologique,* where he wrote: 'the significance of the result of the sequence of actions only depends on the meaning of these actions themselves [. . .] or of their co-ordinations, and not on the properties of the objects' (p. 33).

Much later, cognitive psychology has come to a very similar conclusion. According to Marr (1982), the attribution of a meaning depends, on the one hand, on the nature of the stimuli presented and, on the other hand, on the learner's previous knowledge. Generally speaking, both the Piagetian school and contemporary cognitivists believe that unless a stimulus can be interpreted by the subject in terms of his previous knowledge, it has no significance.

We thus need to find out how often children can 'make sense' of the educational activities they are asked to carry out and, in addition, to know what effect it has on them to be placed in teaching situations where they cannot apply their own schemas of assimilation.

As far as the first of these problems is concerned, we know that, in general terms, people find it quite difficult to identify, accept and envisage new and unfamiliar ways of reacting. More specifically, the history of ideas shows that the modern concept of the child—as a being who has his own particular way of thinking—did not develop to any significant extent before the twentieth century. In the past, children were regarded as 'small adults', differing only in the amount of knowledge they were able to master. It was Piaget who showed that the way in which children think is often different from the way adults think. Unfortunately it seems that Piaget's findings have still not been sufficiently widely accepted to influence teaching efforts. More specifically, teachers still seem to over-estimate the knowledge or cognitive abilities of their pupils and thus plan activities which place them in difficult situations.

In the view of many pedagogues (Kamii & Devries, 1978; Kohlberg & Mayer, 1972, etc.), teaching takes too little account of knowledge that the child has already built up spontaneously. The most obvious example is the teaching of reading. We often begin by identifying a letter to a child, associating a sound with it and then making him practice writing the letter. Once the first letter has been learned, we pass on to a second letter, and so on. Such a method assumes that the child knows nothing about writing. In fact, however, during his first six years, a child in our literate societies has many opportunities to come into contact with writing: illustrated children's books contain commentaries, his parents and his teacher have read stories to him, he has watched his elder brothers and sisters writing, etc. The child has thus already acquired his own ideas of the written word from a multitude of experiences.

Ferreiro (1978) attempts to discover exactly what these ideas are. Using the method of critical exploration dear to Piaget, she shows that all five-year-olds clearly distinguish a drawing from writing. All are convinced that there is a relation between what is said and what is written. Initially, they think that only the actual things or persons mentioned in the spoken sentence are written down. Thus, in the sentence 'John is playing with a ball' only 'John' and 'ball' would be written down. Subsequently, the same children realize that actions must also be written down. In fact, children experience particular difficulties in explaining the spaces between words. Some think that what is written between two blanks is a unit of meaning (John/is playing/ball), while others wonder whether it might not be a syllable.

Ferreiro's research inevitably leads us to wonder what the child himself regards as the purpose of reading after having been taught in a way which totally ignores his initial impressions. In more general terms, we need to examine the risks involved in using a teaching method that ignores the child's spontaneously acquired knowledge. In other words, what goes on in the child's head when the teacher explains an idea that is too complex for him to understand or, a second

possibility, when the teacher introduces a cognitive process which is not compatible with his ways of thinking?

For Furth and Wachs (1974), the answer is obvious: the child loses confidence in his intellectual capacities; he learns to distrust the conclusions to which his own reasoning leads him and finally comes to the conclusion that it is better to avoid having his own ideas. Following the line of inquiry pursued by Giordan and De Vecchi (1987), we might formulate a somewhat less pessimistic—at least, on the face of it—hypothesis: the child builds up his knowledge through contact with the real world and learns from the very beginning to draw on this knowledge to solve everyday problems. In class, he learns to conceal his own ideas and seeks to memorize the knowledge taught him without attempting to connect it to knowledge acquired previously. Consequently, school knowledge becomes a parallel system of knowledge of unquestionable validity since adults are the source of such knowledge and it is they who require him to master it. It is also a kind of knowledge needed to pass tests at school. However, the child will not know how to use this knowledge in other circumstances, since it has been taught to him without any context and with a total disregard for his previous knowledge. Thus, alongside his personal knowledge rooted in action, he will store inert school knowledge which is unusable when it comes to solving real problems.

It may well be that this state of affairs derives from the fact that the school sees its task as being to 'fill a void'. Consequently, it only places 'formal' knowledge alongside practical knowledge, when what it should be doing is to transform vague and poorly articulated knowledge into more explicit and coherent knowledge. This same idea is put forward by Papert (1972) who criticizes schools for organizing what he calls 'dissociated learning' when they should encourage 'syntonic learning', i.e. learning that takes account of the child's own thought processes. At a more basic level, it seems important to us to go back to the concepts of assimilation and accommodation. As Hatwell quite rightly pointed out, as long ago as 1966: 'assimilation occurs whenever the individual incorporates the results of experience into his own personal framework of knowledge' (p. 128). However, the process of assimilation involves more than simply assigning a meaning to a situation; it contains a dynamic that generates the construction of knowledge, since any successful assimilation naturally leads to repetition (reproductive assimilation) which ensures the fixation and, to some extent, the automatization of a schema that has been applied to good effect. In fact, 'since any schema has a tendency to reproduce itself, it incorporates any object that is likely to act as a stimulant' (Piaget, 1970, p. 35). Assimilation is therefore seen as a tendency towards generalization. This property is particularly 'fruitful' since it encourages the enlargement of any given schema and, consequently, of the class of objects that may be assimilated in it (Hatwell, 1966, p. 127-36). It is the failure of an attempt at assimilation which leads to adjustments and accommodations, which, as Hatwell points out 'only occur as a result of a conscious effort on the part of the subject' (1966, p. 127-36) and the creation

of new forms of action or knowledge. Grafted onto the natural tendency towards assimilation, these new processes are immediately inserted into the subject's behaviour: they form part of the way he interacts with his environment and even help to achieve new advances. It is an essential characteristic of the constructive process that any new form of knowledge becomes an instrument of assimilation, i.e. it is first of all repeated several times and then becomes a source of generalization and of further progress.

The concepts of assimilation-accommodation are of crucial importance for the teacher, since they explain in detail the various facets of the constructive progress that needs to be stimulated. In essence, the teacher's task is, from a constructivist point of view, to encourage the child's capacity to assimilate and transform. This can be expressed in more familiar terms as three didactic principles which complement those set out above:

Principle 3. When something new is being taught, the teacher should ensure that the children call up their previous knowledge to help in interpreting the new situation put before them.

Principle 4. In the second stage of learning, he should ensure that the children are given the opportunity to reproduce their new experience and to repeat it often enough to ensure that the new process is stabilized and becomes automatic.

Principle 5. In the third stage the teacher should ensure that the children are able to use their new capacity in new situations that are slightly different from the original context.[10]

On the basis of their research with regard to arithmetical problems, researchers such as De Corte and Verschaffel (1985), Fayol (1989) and Mayer (1981, 1985) have come to a conclusion in line with this last principle when they stress how important it is for teachers to present their pupils with the whole range of categories of problem that can be encountered. This recommendation arises from two research findings:

— De Corte and Verschaffel (1985) have shown, on the basis of a sample of primary classes in the Flemish Community in Belgium, that pupils are more often presented with some kinds of problems than others. In fact, they found no problems of *combination* being presented (see above).
— Mayer (1981, 1985) has shown that the kinds of problems that most rarely occur in books, or among the examples found in the pupils' exercise books, are also the ones with which the pupils deal least successfully.

These findings may be summarized as follows: when practising solving arithmetical problems has been strictly confined to particular kinds of problems, the resulting lack of diversity in the opportunities for learning presented to the pupils create a serious risk that they will only be able to solve a narrow range of problems.

The conceptualization and consequent decontextualization of procedural knowledge

It was this process of assimilation-accommodation that struck Lawler (1981) when he observed on a daily basis how his daughter Myriam dealt with the problems of addition in a non-school context. He also noticed that her capacity to solve a particular problem depended on whether the operation involved money or a more abstract concept. A more general conclusion reached by this American researcher was that the cognitive behaviour of his daughter suggested the existence of *micro-worlds,* blocks of knowledge which can only be called up in very specific circumstances.

This observation is not in itself exceptional. As has been pointed out several times in the preceding discussion of the concept of overall structure, many researchers have observed that a child may be able to mobilize an operatory kind of reasoning in the initial situation, but unable to do so in a similar situation. Lawler's observations (1981) are interesting in that they show that this compartmentalization of knowledge cannot be laid at the door of an artificial approach to the teaching. In other words, we need to recognize that even if we do as much as possible to ensure that pupils learn in meaningful situations, they will not manage to construct an overall system of knowledge which transcends particular situations and, consequently, the teacher will always be confronted with the need to decontextualize knowledge.

This problem of the decontextualization of knowledge has been very clearly posed by Fayol (1989) with regard to the learning of mathematics. He put it in these words: 'The most essential question is to know how the child makes the transition from the accomplishment of localized, specific, unco-ordinated tasks linked with various parameters to a generalized and, inevitably, more abstract form of knowledge' (p. 195). It is Fayol's view—which we share—that the solution is to be found in following up the theoretical model suggested by Klahr (1984). As successive experiences occur, the subject builds up a store of skills or, more specifically, of declarative and procedural knowledge, each element of which is appropriate to a particular class of situations. Either spontaneously or when requested, he is able to carry out processes of analysis and thought on the basis of observed regularities. Klahr speaks of *ruminating about the efficacy of its own processing*. In fact, the subject would carry out what Piaget called reflective abstractions,[11] which would lead him to construct an increasingly general and abstract system of knowledge.

The empirical and conceptual aspect of constructivism strikes us as particularly valuable here, since we need to realize that teaching centred on problem-solving gives priority to the successful accomplishment of tasks rather than to comprehension.

The relationship between action (or rather the ability to deal successfully with problems) and knowledge lies at the centre of Piaget's theory. In fact, Piaget

himself devoted two works to this topic: *La prise de conscience* (1974*a*) and *Réussir et comprendre* (1974*b*). It is this latter work that offers the clearest exposition of his theories. In the conclusion, he reminds his readers of his main hypothesis, tested in many experiments with children of different ages: 'Action gives rise to autonomous knowledge *(savoir-faire),* which is only subsequently conceptualized through conscious thinking' (p. 232). In the same work, Piaget notes that there is always a gap between *successful accomplishment and understanding.*

More fundamentally, it is clear, as far as Piaget is concerned, that the performance of an intelligent action does not necessarily mean there is a conscious awareness in the subject of the rules and anticipations he is applying. Moreover, awareness of the way complex actions are co-ordinated internally is very seldom simple. In most cases, there is conceptualization, that is to say, a reconstruction at another level of the connections involved at the level of action. To put it even more precisely, understanding means building up a structure of concepts such that whatever happens at the level of action is seen as necessary. To attain this level of explanation, where empirical observations are deduced from affirmations taken as premises, we need to construct a model where the links of causality applied in action are expressed as logical implications. As Piaget puts it, 'understanding consists in identifying the reasoning behind things, while accomplishment only involves using them to good effect' (p. 242).

In *L'équilibration des structures cognitives,* which may well be regarded as one of the fullest and best expositions of his ideas, Piaget (1975) distinguishes between three kinds of equilibration. The first is the one referred to above, which results from the dialectical relationship that necessarily exists between assimilation and accommodation. The second involves the co-ordination of different schemes dealing with the same reality. Finally, the third assumes that the subject is attempting to construct an overall representation of a field of knowledge, but is disconcerted by a particular case or a specific item of knowledge which does not fit into the overall structure. Undoubtedly, however, too much importance has been attached to the first form of equilibration and the necessary and deeply integrative character of the two other forms has been too often forgotten. Nowadays, they are seen as hypotheses that can help to understand how pupils attempt to construct increasingly co-ordinated and integrated fields of knowledge. From this strictly didactic point of view, they might lead the teacher to set up situations which make pupils reflect on the nature of the skills that they are drawing on to solve problems which they see as different from those with which they have previously been confronted and, if necessary, to discover their common structure. In short, pupils can only acquire a structure of knowledge which transcends particular situations by dint of applying a considerable effort of reflective abstraction to the areas of knowledge they have mastered. The role of the teacher in this field is probably crucial. Here, as in the case of specific bodies of knowledge, he should not forget that it is for the pupils themselves to construct this overall structure.

We come, therefore, to the final principle with didactic applications:

Principle 6. At a later stage of learning, the teacher will induce the pupils to analyse ways in which they deal with various problems, with a view to identifying the invariants, i.e. the theoretical elements common to different forms of action, etc. He will encourage pupils to be consistent in their efforts to theorize about their approach to problems.

The didactic principles enunciated above would seem to encourage a return to the central feature of Piaget's theory: constructivism. They have no other aim than to help pupils to be able to operate in a context in which cognitive skills are being built up as tools. It is for the pupils themselves to refine, readjust and co-ordinate them and also to reflect on them with a view to a more abstract (i.e. decontextualized) conceptualization. Piaget's work does not totally validate these principles, which should be seen rather as useful lines of inquiry. It is to be hoped that in the future, genuine pedagogical experiments can be designed for rigorous examination of the teaching methods likely to encourage pupils to construct practical knowledge that they can activate to understand the world surrounding them and which they are then able to readjust and conceptualize in response to any disequilibria that they encounter.

Notes

1. A work by Johsua and Dupin (1993) provides a useful review of research on didactics.
2. The expression 'Von Neumann's bottleneck' is used to describe this phenomenon.
3. Those interested in epistemology will have recognized here a process that is well known in the history of science—ideas are discovered, then forgotten, and subsequently rediscovered again from another angle—but they will have some difficulty in suppressing their doubts and anxieties: how many new pedagogical approaches have ended up in the rationalist cul-de-sac which consists in thinking that it is possible to develop transverse skills? Why have the conclusions of constructivist theory been so comprehensively ignored? Epistemologists will wonder anxiously what fate is likely to befall Glaser's contribution in this field, since, although he fully legitimizes the development of specific knowledge, he places this process within a functional context that is likely to be stifled by the intellectual formalism that traditionally reigns in schools.
4. For a more detailed examination of this question, see the article by Lautrey (1985).
5. Greater flexibility is also regarded as necessary by those who are engaged in intercultural studies and who are seeking to combine a constructivist approach with an attempt to identify cultural variations in modes of learning. In this context, it would be both reasonable and helpful to take into account what Dasen (1973) calls 'the relativity of operatory structures'.
6. In his work written with García and published posthumously, Piaget (1983) seeks—significantly—to show that it is possible to reach the same conclusion by different paths (p. 28-29). In a prior work also written with García, Piaget (1971) agrees that the child has only an occasional interest in 'arranging or classifying things for the sake of it'. He points out that 'the general function of operations is to act on the real world by building up frameworks and structures that will enable it to be assimilated'

or understood. Thus, 'the structure in question (in a particular problem) is a form, and as such is constructed through the action taken by the subject to structure a given content' (p. 20-26). Transposed into the pedagogical context, this represents a warning against the danger of learning concepts or mathematical structures independently of any functional context. For Piaget, logico-mathematical structures gradually become differentiated from their underlying contents. They only acquire a psychological existence of their own when the subject has reached the formal stage for a specific field of knowledge.

7. In his article *Stades et différences,* Lautrey (1985) describes the different stages in Piaget's thinking on this question of stages and the concept of *overall structure.*
8. It is worth noting in passing the extent to which the positions taken by Gagné changed between 1977 (the date of publication of *The conditions of learning*) and 1985 (publication of *The essential of learning for instruction*). The same could be said of Glaser. In fact, both of them have abandoned behaviourist positions in favour of a cognitivist approach.
9. Writers aware of the evolution of psychological theory will have noticed that we place references to Piaget's work alongside the contributions of contemporary cognitivism. That may appear surprising, since it is the fashion nowadays to contrast these two conceptual systems. However, although aware of the dangers of excessive eclecticism, we think it more helpful to try to combine these two currents of research. The former seeks mainly to understand the process whereby knowledge is built up in an epistemological context, while the latter is more geared to its use.
10. Glaser (1986) makes proposals somewhat similar to these three principles in concluding his examination of the recent contributions of American cognitive psychology to education.
11. According to Piaget himself (1961), reflective abstraction 'consists in drawing from a system of actions or operations at a lower level certain characteristics which are then reflected (in the almost physical meaning of the term) onto actions or operations at the higher level, since it is only possible to become aware of previous construction processes through reconstruction at a new level' (p. 203). In other words, 'reflective abstraction' operates through reconstructions which transcend, and at the same time integrate, previous constructions' (p. 203).

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LEARNING—THE DRIVING FORCE OF DEVELOPMENT

Ludmilla F. Obukhova

There are in psychology two fundamentally opposed conceptions of learning and development: Piaget's view and that of Vygotsky, for whom learning precedes development. In accordance with his hypothesis of the 'proximate development zone', mental development is measured in the light not only of the self-sufficiency of the child's activity, but also of its potential for co-operation with the adult. According to Piaget, learning follows the spontaneous intellectual development of the child and can be successful only to the extent that the experience of the child is applied. In order to bring out his thinking more clearly, Piaget draws a distinction between two meanings of the notion of learning. Broadly speaking, the main aim of learning as thus conceived would be to verify the passage from one development stage to the next through a study of the underlying mechanisms of the acquisition of new knowledge. According to the psychologists of the Genevese school, the learning experience is not only of importance for genetic psychology, but may also be used in solving some difficult epistemological problems.

The mechanisms of learning

In Piaget's system of genetic psychology, the acquisition of the principle of

Ludmilla Filippovna Obukhova (Russian Federation)
Doctor of psychology, lecturer in child psychology in the Department of Developmental Psychology, Moscow State University. She has studied the mental development of the child in the context of Lev S. Vygotsky's concept of cultural history. Her writings include: *Stages in the development of thought in the child* (Moscow, 1972); *The views of J. Piaget: pros and cons* (Moscow, 1981); jointly with G.V. Burmenskaya and A.I. Podolsky, *Modern American developmental psychology* (Moscow, 1986); co-author with I.V. Shapovalenko, *Forms and functions of imitation in childhood;* co-author with S.M. Churbanova, *Divergent thinking development in childhood* (Moscow, 1994); *Child psychology: paradigms, facts, problems* (Moscow, 1995).

conservation (invariance, constancy) marks an important stage in the intellectual development of the child. Conservation should be understood as meaning that an object or a set of objects are acknowledged to be unaltered in their composition or in any other physical parameter, whatever changes there may have been in their shape or their external arrangement, provided that nothing has been removed or added. According to Piaget, acquisition of the principle of conservation is the psychological criterion of the emergence of a fundamental logical characteristic, viz. reversibility, testifying that the child has made the transition to a new level of thought, the concrete-operational phase. It is also the necessary condition for the child to form scientific ideas, which explains why it is of interest to analyse how the child arrives at an understanding of conservation, and upon what it is based.

Scholars of many countries have tried to develop the notion of quantitative invariance (conservation) and an understanding of the logical relations between the part and the whole in the pre-school child.

In accordance with a hypothesis accepted by a number of psychologists, and especially by Smedslund, the child may acquire the idea of conservation by observing the repetition of external confirmations. As has been experimentally verified, confirmation through counting or weighing and through the reaction of the experimenter (favourable or unfavourable) produces the result that the child gradually comes to compare physical quantities better and with greater precision, although their logical relationships remain beyond his/her grasp. With learning of this type, the child merely acquires empirical knowledge without appreciating a logical principle. A further experimental conclusion is that learning based on the direct method and outside confirmation or on simple training appears incapable of leading to real knowledge, although it is an all too widespread practice in education.

The experiments carried out by Morf in teaching the child to understand class-inclusion relations led to verification of the hypothesis that the source of logical operations is to be found in the co-ordination of the subject's actions. In order to induce the formation of a new logical structure, the experimenters accepted as an exercise other operations that had already been mastered by the subjects and were directly connected with the structure to be acquired. Such training is in fact conducive to an understanding of the logical relationships and speeds up the acquisition. Be that as it may, these experiments do not always show clearly how the transition from the first operations to the others is effected, why the subjects do not give the right answers to all the tests put to them, or why those who are successful do not advance immediately through to the end.

Inhelder and his collaborators have verified the hypothesis that the main condition for the formation of the logical principle of quantitative conservation is indeed that a situation of conflict arises. The child finds himself/herself obliged to relate the changes in the outward form of the object to its size in order to distinguish between the invariant parameter of the object and its non-essential variable parameter. What is emphasized in this type of learning is the part played by the situation with its apparent contradiction. This situation should lead the child to

resolve the contradiction and, in the final analysis, to make the transition to the higher thought stage. Not all the subjects in these experiments gave the right answers to all the tests put to them, but that is not surprising, given that rational creation of a situation will not suffice for the development of true logical knowledge unless the child's activity is organized.

It is another hypothesis of the Genevese psychologists (especially Perret-Clermont) that the intellectual development of the child ought not to be envisaged in a social vacuum, in the sense that it is a process that should be put back into the context of social interaction. Succinctly, it is the social conflict and its resolution that stimulates development of the child's cognitive abilities. The collective action of a number of children is subordinated at a certain stage in development to the resolution of their antagonisms, and the resolving of these socio-cognitive conflicts leads to the emergence of new intellectual structures. According to this interpretation, the formation of logical structures is limited, because the only way to direct the development process is by bringing into play other pre-logical or logical structures (minimal intellectual ability) previously acquired spontaneously.

Action and development

The facts noted during the progressive acquisition of intellectual operations and ideas (Galperin) cast new light on the problem of the relationships between 'learning and development'. We, for our part, study these new potentialities at one of the clearest turning-points in the intellectual development of the child, namely at the time of transition from late pre-school age to early school age.

Although Galperin, like Piaget, sees action as lying at the origin of the development of thought, their understanding of it differs.

According to Piaget's theory, thought is a system of operations, in which the operation is an action that has been transposed to the intellectual level, been simplified, made reversible and co-ordinated with the others to form a whole that constitutes a system. Piaget defines the action in terms of the level at which it is carried out (internal, external), its mobility (irreversible, reversible) and its degree of completion (it may be reduced, executed symbolically, or developed, realized through real objects). Piaget, however, does not reveal either the psychological mechanism of the action or its psychological structure, but merely emphasizes its physical and logical aspects.

Galperin, on the other hand, distinguishes two basic components—orientation and execution—in each concrete action of the subject. The action that the child has to master constitutes an objective process, the content of which is preassigned; the model of the action and of its product is an objective representation and its plan also has an objective existence. These are the three essential elements that maintain the orientation of the action and which, directly or indirectly, must find their expression in it, because it is the psychological mechanism that impels the action, and it is upon its content that the success and general quality of the action naturally depends.

The subject's action is thus characterized by a complex interaction of orientation and execution, which does not in itself suffice to account for it. This remark applies in particular to Piaget, who does not envisage anything in the action other than its execution—or, to be more precise, he treats it globally, without distinguishing between the psychological and the concrete content. Consequently, he does not see any possible interpretation of the development of the action other than its conformity to formal logical structures, and does not elucidate the strictly psychological mechanism of the action.

This difference of outlook on action leads to other differences regarding thought, its formation, and the stages and mechanisms of its development.

Measurement and conservation

Arising from our analysis of the ways in which children of pre-school age solved problems of conservation, we concluded that their replies and characteristics stem from the fact that the child perceives the different properties of the object, but does not separate them, and forms an 'overall' assessment of it in accordance with some particular dominant character. The reason for this, moreover, is that the child has no way of making the transition from direct assessment of orders of magnitude to their measurement and assessment from the results. That is why calculation carried out by a child does not always fulfil its purposes. Having counted objects, the child does not draw a conclusion from the result, and will even forget the number on encountering an image that specifically suggests something else. The idea of measurement, which arises spontaneously and is the tool that basically serves to establish the invariance of a dimension when the outward configuration is modified, is scarcely developed at all in pre-school-age children.

A distinction must be drawn between measurement as a means of distinguishing between the parameters of an object and taking one of them as a starting-point to demonstrate its invariance, and another category of means used to record and determine what has been measured. Because these indicators are connected with measurement, they are a source of information on it which enables the child to make a dimensional comparison that is already mathematical, even if pre-numerical. We therefore adopted the initial hypothesis that it is possible to appreciate the size of an object in terms of the parameter being sought by means of measurement and its indicators, and then to note the conservation of the quantity in Piaget's exercises.

In seeking to verify this hypothesis, we conducted an experiment with fifteen pupils (5 and 6 years old) in a rural nursery school. Other researchers have since repeatedly reconstructed the method by which the idea of conservation is formed in pre-school-age children.

The experiment in question showed that none of these children understood the principle of conservation: in making their judgements they referred only to the external characteristics of the objects.

Attempts to introduce measurement along with Piagetian exercises ended in

failure; the child succeeded in making the measurement, but failed to grasp the significance of the result being guided by its direct image. As in the previous case, the child estimated the dimension from the image, making a judgement that was immediate and undifferentiated.

For example, we gave a child two bottles half-filled with lightly coloured water. Using a small glass, the child had to measure the amount of water in each bottle and establish that the content was four measures. The child said that the amount of water was 'the same'. The experimenter then stoppered one bottle and turned it upside down. It was strikingly apparent that the water level was higher in this bottle than in the other one. Despite the measurements previously made, the child said that there were four measures in the bottle in the original position, but 'seven' in the upside-down bottle. When we asked one subject why he thought that this was so he replied 'I reckoned up in my head'. In some instances the children had made measurements and clearly remembered that three glasses of water had been poured into each of the two bottles, but they could not bring themselves to realize that there was the same amount of water in each. According to them, the number of glasses was the same, but the amount of water was not. In the end the correct answer was obtained from some children who had understood from measuring that the quantity of water had been conserved in the bottles, but it was extremely difficult to hold their attention on the measurement results. The slightest change in the experimenter's tone of voice when asking the child to explain his reply led to a change of mind by the child. Following our explanations, there were admittedly also subjects who quickly learned to turn to the measurement results and to give correct replies. Even so, we were unable to establish why or how the transition had been made in those cases.

Clearly, what was firstly needed was to create a new mediate mode of thought—at the external and then at the internal level—to reinforce it and only subsequently to confront it with reality.

In order to teach the children how to compare sizes indirectly, we had to devise problems that could be solved only by the use of measurement and auxiliary means. The act of learning how to carry out mediate evaluation of the various parameters of an object was a three-stage operation.

We began with acquisition of the ability to use sizes by presenting the child with cutouts stuck on a card in a random order. There were cutouts of two different types on each card, and the child had to say which were the most numerous. Because the cutouts were stuck on the card it was impossible for the child to place them side by side. As there were far more cutouts than the child could count, the only way to solve the problem was by using indicators that children could handle with ease. The experimenter gave each child a supply of solitaire counters and rods. The child put, for example, a rod on each 'fox' and a counter on each 'duck'. The effect was that rods made the child think of 'foxes' and counters of 'ducks'. The experimenter then presented the child with a card on which were two square boxes and a long row of double arrows. The child could put the 'duck' in the upper box and the 'fox' in the lower one, i.e. had to place the coun-

ters reminiscent of 'ducks' in the upper row and the rods reminiscent of 'foxes' in the lower one. Having established a one-to-one relationship, the child was able to give the right answer to the question.

The second stage consisted of learning how to compare two objects by means of a third. It is common knowledge that children like to take part in discussion on the direct comparison of two magnitudes, but we were setting up an exercise in which direct comparison of the dimensions of the figurines was impossible. In order to determine which of two glued-on cutouts (two keys, two carrots etc.) was the larger, a third object had to be used—a strip of coloured paper—which we showed the child how to handle. The child had to cut a length of this strip that was exactly the length of one of the cutouts and then place it on the other to see whether it was larger or smaller.

The comparison was confined to the dominant characteristic, because what mattered initially was to teach the child the mediate estimation technique. This third element isolates the parameter under consideration and indicates its size, but it has the drawback of appearing to be in itself an independent concrete object, rather than an instrument that transforms the measured size into a quantity. This limitation makes comparison through a third element a special atypical measurement case. It was for that reason that the following stage was devoted to teaching the child how to make use of a clear and precise measurement. What the child had to do on this occasion was to compare, for example, the length of two 'stepped' lines or two 'paths', using a small strip of paper on which the length measured with markers was noted.

We also set up exercises involving volumes, areas and weights. In one, the children were shown clearly how to proceed. What was required was to use a small glass to measure the grain content of two boxes of different dimensions, with the object of giving the larger quantity to birds. In another the subjects weighed a large pencil and subsequently a small nail on beam scales and were astonished to find that objects that differed so much in size weighed the same.

Having taught the children to use a measuring instrument or auxiliary means (indicators) to estimate dimensions and always to use them in the exercises, we turned to the different properties of an object, once again employing measurement. We gave the child real objects (two rods, two books, some cotton and a stone, etc.) and asked him/her to determine those dimensions (length, breadth, height, area, weight) with respect to which these objects were alike and those with respect to which they differed.

Let us here consider only one example out of many. Subject: Natasha N. (five years old). Experimenter: 'Which is bigger, the nail or the pencil?' Subject: 'The nail is small and the pencil is big. The nail is made of iron and the pencil is like a stick. The pencil is bigger than the nail.' Exp.: What would we use to measure the length of the pencil?' The subject chooses the means of measurement and measures the length first of the pencil and then of the nail. Exp.: 'What is it about the pencil that is bigger than the nail?' Subject: 'The length.' Exp.: 'These objects have another property that we can find out on the scales.' Subject: 'Their weight.'

She weighs the pencil and the nail quite adequately using similar coins and enters her findings on the form. Exp.: 'How is the pencil more than the nail?' Subject: 'In length'. Exp.: 'And now, how is the nail more than the pencil?' Subject: 'In weight.' During the experiment the children were able to give the right answer to the question 'Which is more?' that had been put to them purposely without specifying in what respect it was more. This they did by indicating the parameters in respect of which one object differed from the other and those in respect of which they were alike (the same).

Thanks to these measurement operations, the children started to distinguish the various parameters of the objects without difficulty and to estimate their dimension not globally, but in terms of a well-defined property.

The use of measuring instruments and auxiliary means (indicators) makes it possible to present the object with a different appearance. To begin with, the child is confronted with objects as initially presented to it. The use of measuring instruments and markers leads to the construction of a new model of the relations between these objects, as expressed in a definite ratio between the indicators. This schematic representation of the essential relations between the objects is the outward manifestation of what will subsequently be the inner framework of the child's reasoning.

The mode of reasoning thus formed was then transposed to Piagetian (and similar) exercises including the parameters of length, area, interval, weight and volume. There was a noticeable divergence in the initial exercises between the opinion arrived at by guesswork based on appearances and reasoning based on measurement. What had happened was a splitting of the same impression: the children had appreciated what was merely seeming and what actually existed. Despite the experimenter's 'provocative' and disconcerting questions, the subjects justified their replies by relying on measurement. Less frequently they invoked the rule 'nothing was taken away or added'. They also knew how to justify the invariability of the amount of matter from reversibility—because they had in a sense tackled the question of conservation experimentally.

Let us take the example of Sasha M. (6 years old). Experimenter: 'I have put some grain into the bottle. How many measures of grain do you think there are in it?' Subject: 'Two measures. I can't tell, I don't know how much grain there is. I haven't measured it.' He proceeds to measure the grain in one bottle. 'There are four measures of grain in this bottle.' He measures the grain in another bottle. 'There are four measures in this one as well. The bottles are completely the same. There is the same amount of grain in the bottles.' The experimenter turns one bottle upside down. Subject: 'There are four measures of grain here again, and there. There is the same amount of grain in all the bottles.' Experimenter: 'Why were there four measures of grain and why are there still four?' Subject: 'Because no grain has been taken out.' Exp.: 'Where is there more grain?' Subject: 'There is the same amount in them all.' Exp.: 'How did you find out that?' Subject: 'By measuring.'

When we set similar exercises to other pupils from the same class of the nur-

sery school who had not been a part of our experiment we got typically Piagetian replies. These pupils had no criteria from which to reason apart from outward appearance. In the experimental situation, recourse to measurement remained a proof of invariance for the children. What is of interest is that the children were resorting to measurement even when, for example, it was glaringly apparent that two rods were of the same size. It is quite evident that this measuring enables the child to isolate the relevant parameter—the length of the rods in the case in point.

Let us take another example. Lena K. (6 years old) takes the rods and examines them. 'I must measure that one.' She measures its length. 'This rod is eight measures long. That one will also be eight measures long, because these rods go together. They are equal. If we cut off a bit, they would be unequal, but as they are, they are equal.'

What emerges from these experiments is that in the first exercises the child was, as a rule, still reasoning on two levels. Initially, when faced with the question 'Which is more?', the child behaved in a Piagetian way, and, when asked 'How can we know which is more?' (in length, volume or area), said that these quantities had to be measured and, having done so, saw the invariance of the property concerned and then explained it by saying 'Nothing has changed, because we haven't taken anything away or added anything' or 'There is as much, because if we did it again as it was, there would be as much'.

We had not taught these arguments to the children; they already knew them, although before our intervention all their significance and persuasiveness would occasionally be lost in the face of a concrete, striking image that appeared to contradict them. What had been necessary, at the beginning, was to separate out the various properties of the objects, then to stipulate which property was concerned in the exercise, and then to establish the invariance of that property (by measuring), after having presented the object both in its initial form and as schematicized and transformed, and lastly to reinforce this mediate reasoning scheme—only after which could arguments and reasoning of this type acquire stability and psychological force when confronted with the immediate image of things and become a logical principle of the child's thought. Thereafter, the new mediate scheme became the principal one and, as the children themselves said, what 'seemed to be' was not the same as what 'actually' was. We now understand the basis of that distinction: at the beginning of our teaching the child was already taking the essential relations between objects from the image with which it was confronted and embodying them in a spatial framework. This second mediate scheme rapidly established itself and replaced the first one, not only in judgement but in perception.

At the beginning the first exercises were carried out in an externalized and well-materialized way: the quantities measured before and after the alteration were noted by means of indicators, the one-to-one correspondence of the two quantities was established and the invariance of the size of the property under consideration was deduced. Fewer measuring operations were subsequently carried out. Whereas the child initially had to measure a number of parameters

before and *after* the configuration of the object was *modified*, thereafter the parameter was measured only *before* the change in order subsequently to provide a logical justification of its invariance, its conservation despite the various changes in the form and disposition of the objects. The child was later able to dispense with any measurement: it was sufficient to establish by guesswork that the objects were similar with respect to a specific property; the child could then assert, relying solely on logic, that there was real conservation of that property.

The point ultimately reached corresponds exactly to the situation of subjects put through Piaget tests after having mastered the principle of conservation. All the conservation exercises that we tried, which took in many physical parameters (length, weight, volume, area) and objects that were different either in structure (discrete and continuous quantities) or in material (lumps of modelling clay, paper cutouts), were correctly carried out by the children, who always justified their replies.

Nonetheless, as all the exercises that we set were concerned with the conservation of equality, the children might, just by realizing that the situation was being repeated, have contented themselves with answering mechanically, often in the same terms. We therefore decided to ascertain that the children were well able to appreciate the conservation of inequality. Use was made, to that end, of exercises on the conservation of inequality involving amounts of liquid in receptacles of the same shape and the weight of two lumps of modelling clay. On each occasion the object transformed was the one that was slightly smaller than the other but which, on alteration, 'grew larger' with respect to a highly obvious external parameter (the height of the water column, the length of the sausage-shaped lump of modelling clay). The fact that the subjects always justified their replies in these exercises demonstrated that they had a proper understanding of the situation.

That was the case, for example, of Serezha S. (six-years old). Experimenter: 'Find the heavier lump.' The subject weighs the lumps on the two scale pans. 'That's the one.' When asked to do so by the experimenter, the child flattened the lighter lump into a biscuit. Exp.: 'Which is heavier, the lump or the biscuit?' Subject: 'The lump. If the lump is put on the scales, it will go down and the biscuit will go up. The biscuit was a lump, and when I put it on the scales it went down. I put on the other lump and it went down and the biscuit went up. The lump is heavier than the biscuit.' He picked up the biscuit again and rolled it into a lump and then into a sausage. 'The lump is heavier. When we began measuring, it was that ball that was heavier than the other.'

We also verified the stability and durability of the ideas formed by the children. To that end, we set them new conservation exercises one month after the learning. They did not show any sign of having forgotten. On the contrary, we noted that the children did these exercises with greater facility, and said that they were 'easy'.

At the same time, it should be noted that transfer of the principle of conservation to new exercises was not hampered either by the material or by the parameter given in the question. We did not note any delay or 'shift' (to use

Piaget's term) in the formation of the idea of conservation. The child answered correctly and concisely (in accordance with the rule: 'nothing was taken away or added') in the exercises on conservation of length, weight, volume and area, and on the total quantity of discrete and continuous size.

Mediate and immediate thought

The attitude of our subjects, on the one hand, and of the pupils in the same class who had not taken part in the experiment, to these exercises enabled us to note the emergence of two fundamental modes of knowledge—the instrumental, mediate mode and the immediate mode. The results of our experiment justify the view that, provided the process of the assimilation of new knowledge has been carried sufficiently far forward in children who are in the last year of nursery education, a start can already be made on instilling into them the rudiments of a truly scientific approach to the phenomena of reality. One can organize the formation of truly scientific ideas in the child by judicious use of the tools that have been traditionally and ubiquitously employed and are available to children in the community (measures, yardsticks, concepts), and also by using auxiliary means (indicators, one-to-one correspondence, reckoning).

As our experience has shown, the subject is led, through action, to distinguish in the outward image of things between their appearance and the essential relations that are concealed beneath it. This distinction is of great importance for the formation of the inner level of thought. The possibility of distinguishing the essential relationship inherent in the object arises because of the availability in action of a tool, a measure or other auxiliary means. In Piaget's view, action is aimed at the manipulation of objects, in the course of which the child has recourse to action that he/she carries out and which constructs the object. In our view, the essential point is that the child finds his/her own bearings through his/her own action. Consequently, action does not immediately give rise to a construct of reality, as Piaget thought, but to an elucidation of the structure of reality, and it is that structure which is reflected in the child's consciousness.

When we teach a child how to use an objective criterion to isolate parameters, and how to convert them into mathematical quantities and establish a one-to-one relationship between them, the child's initial idea of things is thereby modified. The child begins to draw a distinction between what seems to be and what actually exists. In so doing, the child passes beyond the limits of its immediate experience and sees opening up before it for the first time the possibility of a truly scientific understanding of the world.

The transition to operative thought

As has been shown in the work of Piaget, mastery of the principle of conservation is evidence that the child has made the transition from pre-operative to operative thought. We developed the idea of conservation of the amount of various physical

magnitudes in our subjects by applying the method of the development of operations and intellectual ideas developed by Galperin. This method enables us to take a new look at the process of intellectual development in children.

What emerges from these experiments is that pre-operative thought and thought capable of concrete operations are not two stages of a single and even lengthy period in the intellectual development of the child, in which Piaget sees the development of the operative structures of intelligence as an uninterrupted process, but that they are actually two essentially different types of thought. The transition from one to the other in the child is not, in our view, a harmonious progression preparing the way for the carrying out of concrete operations (as it is for Piaget), but a qualitative leap from pre-scientific thought to the first truly scientific thought.

Our research prompts us to formulate a hypothesis on the nature of this transition.

Its first major feature is the *change in the child's attitude.* At the pre-scientific level of thought, the child arrives at a judgement that is immediate and 'egocentric'. In the setting of our experiment, we taught the children to assess objects by measurement, which serves to give material expression to an objective attitude to things. By accepting and making use of measurement, the child, *ipso facto,* adopts a new attitude for assessing things.

The danger is that measurement will be no more than a technical means for the child, in which case there may well not be transition to another attitude. Just how measurement, in ceasing to be a technical means, is transformed, for the child, into an objective assessment criterion and also provides the child with a new intellectual view of things, remains an open question. We prompted this transition by creating conditions in which the child had to use measurement to solve a problem, being unable to act in any other way or to pass an immediate judgement on the situation that would come naturally. Necessity compelled the child to have recourse to measurement.

The transition from the egocentric to the objective attitude that finds its expression in recourse to measurement, is not equivalent to Piaget's concept of decentration. It is through centration that the subject isolates indicators in an object that permit its assessment. The child proceeds to other indicators for empirical assessment through a new centring, which is why such decentration does not invariably lead to a change in attitude or to modification of the initial view of the world.

Intellectual attitude is something that is more general and more essential than centration and decentration. It is a position taken up by the subject in the system of beings and things, in the psychological situation created by the socially significant problem that the subject is trying to solve. It is, at the same time, a system of criteria that has been spontaneously built up by or inculcated into the subject to enable the situation to be analysed. It is far from being a matter of indifference whether the child examines the possible criteria for the assessment of phenomena empirically, from its own subjective standpoint, or considers things

from the standpoint of criteria elaborated by society. In the first case, transition from one element of the situation to another might not alter the picture of the world that the child has formed, while in the second case that picture will be qualitatively transformed.

The second and highly important effect that characterizes the transition to scientific thought is *modification of the picture of the world.* As a rule, things appear to the child to be as they immediately seem to be. Even should there be something behind the external aspect of the object, it remains mysterious and obscure to the child. Before we intervened, the subjects always took the appearance to be the reality; subsequently they began to distinguish between appearance and actuality.

In the course of our experiment, the children arrived at a new representation of things by making use of measurement to analyse the object and by using indicators to establish them. They were able to pinpoint particular properties of the object by making various measurements and to divide up an entity that they had previously seen globally. Furthermore, the fact that each property had been quantified by measurement enabled them to make precise comparisons of objects.

By working in this way, the children themselves became experimenters so as to find out things they had previously not known. The use of measurement became for them a new method for studying their environment. We have had occasion to note a kind of behaviour that might, at first sight, seem paradoxical: the children set about measuring the length of two rods even when they were put side by side and could clearly be seen to be of the same length. This behaviour is characteristic of children who have not yet learned to distinguish clearly between appearance and reality, and by making the measurement they stimulated this differentiation.

Transition to the first scientific representation of things is possible only in a child who has acquired the new tools of thought constituted by the intellectual instruments needed for analysis that serve for the study of each specific case at the same time as they reveal the essential aspects of things. *The development of a new intellectual instrument* is the third characteristic of this transition.

In the setting of our experiment, we taught the children to use a spatial schema to represent the structure of the essential properties of the object. Only by subjecting the object to measurement and its auxiliary means was it possible to create a new model of it: its structure had been detached from the initial, global and undifferentiated picture, and its essential relations given material expression by the ratios established between the indicators. In this case the child was simultaneously confronted by the initial aspect of the object and the schema arrived at following its transformation. This schematicized representation of the essential relations of the object served to give concrete external expression to what subsequently became a new tool of thought for carrying out Piagetian exercises.

The spatial scheme is, admittedly, not yet in itself an intellectual tool, but becomes one only when the child creates it, through action, by measuring the object, and uses it to analyse new objects. This schema enables the child to find

his/her bearings in new objects and is of assistance in elucidating their structure.

Children succeed in carrying out Piagetian exercises only so far as they organize themselves to find their bearings in things. We were not satisfied with postulating the existence of this trend, but built it up. It is not, in our view, the spontaneous development of operative structures that leads to the understanding of things but, conversely, this understanding that we, the adults, know how to organize in the process of teaching the child, that leads to the formation of logical operations.

We constructed our experiment in accordance with the method of the systematic formation of operations and intellectual ideas. This method enabled us to instil new knowledge into the subjects, but it also revealed something else to us, namely, how the elements of scientific thought take shape. It is on that basis that we are able to assert that the building up of knowledge is not its only purpose because it is also, under certain conditions, a method for the study of intellectual development upon which the learning capable of being the driving force of development may be built.

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PIAGET'S THEORY AND THE TEACHING OF ARITHMETIC[1]

Constance Kamii

Piaget studied the nature of human knowledge scientifically and made a fundamental distinction among three kinds of knowledge—physical knowledge, social (conventional) knowledge and logico-mathematical knowledge. This distinction enables us to view arithmetic in a new way and to teach it very differently from traditional instruction. In this paper, I will first clarify the nature of logico-mathematical knowledge. I will then show that a new theory about the origin and construction of mathematics leads to educational goals, classroom practices and an approach to evaluation that are very different from traditional ones. In the final part of this paper, I will explain why, in my opinion, traditional instruction is not only unnecessary but also harmful to children's development of numerical reasoning.

The nature of logico-mathematical knowledge

The distinction made by Piaget (1965, 1971) among the three kinds of knowledge is based on their ultimate sources and modes of structuring. *Physical knowledge* is knowledge of objects in external reality. The colour and weight of a block are examples of physical properties that are in objects in external reality and can be known empirically by observation.

Examples of *social knowledge* are holidays, such as Mother's Day, written and spoken languages, and the rule of saying 'Good morning' under certain circumstances. While the ultimate source of physical knowledge is partly in objects,

Constance Kamii (United States of America)
Constance Kamii studied under Jean Piaget, Barbel Inhelder and Hermina Sinclair in the late 1960s and during the 1970s and subsequently developed a pre-school curriculum based on Piaget's theory. She later extended this work to mathematics education in the first three grades and is now working at the fourth and fifth grade level. She has taught at the University of Illinois, Chicago, and the University of Geneva, and is now a professor of education at the University of Alabama, Birmingham, in the United States.

the ultimate source of social knowledge is partly in conventions worked out by people. The reason for saying 'partly' is clarified shortly.

Logico-mathematical knowledge consists of relationships created by each individual and is the hardest kind to understand. For instance, when we are presented with a red block and a blue one and think that they are *similar*, the similarity is an example of logico-mathematical knowledge. Almost everybody thinks that the similarity between the blocks is observable, but this is not true. The blocks themselves are observable, but the similarity between them is not because the similarity exists neither *in* the red block nor *in* the blue one. If a person did not put the objects into this relationship, the similarity would not exist for him or her. The ultimate source of logico-mathematical knowledge is thus in the mind of each child. Other examples of relationships an individual can create between the same blocks are *different, the same in weight* and *two.* Mathematical knowledge such as 2 + 2 = 4 and 3 x 4 = 12 is constructed by each child by making relationships out of previously created relationships.

It was stated earlier that the source of physical knowledge is only *partly* in objects. Piaget's reason for saying 'partly' is that a logico-mathematical framework, or a classificatory framework, is necessary even to recognize a block as a block. Classification is also necessary to think about the colour of an object and to recognize the colour as being blue. Without classification, it would be impossible to construct physical knowledge. Likewise, it would be impossible to construct social knowledge without a logico-mathematical framework. To recognize a certain word as a 'bad word', for example, the child has to categorize words into 'good ones' and 'bad ones.'

YOUNG CHILDREN'S REACTIONS TO A NUMBER TASK

The best way to clarify the nature of logico-mathematical knowledge and, more specifically, of number concepts is with a task devised by Inhelder and Piaget (1963). In an individual interview, the child is given one of two identical glasses, and the interviewer takes the other glass. After putting thirty to fifty beads (or beans, buttons, etc.) on the table, the interviewer asks the child to drop a bead into his or her glass each time she drops one into hers. When about five beads have thus been dropped into each glass with one-to-one correspondence, the interviewer says, 'Let's stop now, and you watch what I am going to do.' The adult then drops one bead into her glass and says to the child, 'Let's get going again'. The adult and the child drop about five more beads into each glass with one-to-one correspondence, until the interviewer says, 'Let's stop'. The following is what has happened so far:

Adult: 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1
Child: 1 + 1 + 1 + 1 + 1   + 1 + 1 + 1 + 1 + 1

The adult then asks, 'Do you and I have the same amount, or do you have more, or do I have more?'

Four-year-olds usually reply that the two glasses have the same amount.

When we go on to ask, 'How do you know that we have the same amount?' they explain, 'Because I can see that we both have the same amount.' (Some 4-year-olds, however, reply that they have more, and when asked how *they* know that they have more, their usual answer is 'Because'.)

The interviewer goes on to ask, 'Do you remember how we dropped the beads?' and 4-year-olds usually give all the empirical facts correctly, including the fact that only the adult put an additional bead into her glass at one point. In other words, 4-year-olds remember all the empirical facts correctly and base their judgment of equality on the empirical appearance of the two quantities.

By age 5 or 6, however, most middle-class children deduce logically that the adult has one more. When we ask these children how they know that the adult has one more, they invoke exactly the same empirical facts as the 4-year-olds. (Children in some environments develop more slowly than others, but they sooner or later all come to say that the adult has one more.)

No one teaches 5- and 6-year-olds to give correct answers to these questions. Yet children all over the world become able to give correct answers by constructing numerical relationships through their own ability to think. This and countless other Piagetian tasks demonstrate that children construct number concepts from within. When the child becomes able to make numerical relationships, the one-to-one correspondence in the task changes from being mostly empirical, physical knowledge to becoming both physical and logico-mathematical knowledge.

Educational goals, classroom practices and evaluation

On the basis of Piaget's theory about the origin and nature of children's numerical reasoning, I hypothesized that it should be possible for children to create their own arithmetic. This hypothesis was amply confirmed as can be seen in Kamii (1985, 1989*a*, 1989*b*, 1990*a*, 1990*b*, 1994). This hypothesis also continues to be confirmed in fourth and fifth grades, where computation involves fractions, decimals and percents. I will now discuss the ways in which a new theory gives rise to different kinds of educational goals, classroom practices and methods of evaluating the results.

GOALS IN ARITHMETIC

If children develop number concepts and logico-mathematical knowledge through their own ability to think, the goals in arithmetic must be that they invent their own procedures for solving problems and construct a network of numerical relationships. These goals are different from the traditional ones that focus on correct answers and the writing of mathematical symbols. They are also different from those of getting children to internalize 'addition facts', 'multiplication facts', and the rules of 'carrying', 'borrowing', etc.

The traditional aim of getting children to internalize 'facts', such as 'addition

facts', comes out of beliefs that do not differentiate between logico-mathematical knowledge and physical knowledge. Facts are observable, but neither numbers nor sums are observable as stated earlier. Therefore, there is no such thing as an 'addition fact.' Furthermore, sums are not learned by *internalization* from the outside. They must be *constructed* by each child, from the inside.

The traditional goal of getting children to internalize rules such as those of 'carrying' and 'borrowing' is rooted in empiricist thinking that does not differentiate between logico-mathematical knowledge and social (conventional) knowledge. From the standpoint of logico-mathematical knowledge, it makes no difference whether the child adds 38 and 25 by adding the ones first or the tens first. However, the conventional rule given in almost all textbooks makes children add the ones first. We will see shortly that when children are encouraged to do their own thinking, they invariably add the tens first.

An example of the network of numerical relationships we want children to construct can be seen when first graders begin to do 5 + 6 by deducing that because 5 + 5 = 10, then 5 + 6 = 10 + 1. Many first graders can give an immediate answer for 5 + 5, but when asked about 5 + 6, they do: 5 + 1 + 1 + 1 + 1 + 1 + 1. This usually happens when children have not constructed a network of numerical relationships that enables them to think about 6 as 5 + 1, 4 + 2, 3 + 3, 2 + 4, 1 + 5, 7 – 1, 8 - 2, 9 - 3, 10 - 4, 2 x 3, or half of 12, etc.

CLASSROOM PRACTICES

The method does not use any textbooks because textbooks utilize associationist-behavioristic principles for the mastery of superficial and narrow objectives through repetition and reinforcement from the outside. Instead, we use three kinds of activities: numerical reasoning in daily living; group games; and discussions about problem-solving.

The use of situations in daily living

If children have been constructing number concepts through daily living since infancy, it follows that we should use the same kinds of situations in school. Taking attendance, voting, collecting money, distributing notes to send home, and dividing the class into small groups are examples of situations the teacher can use to encourage pupils to think numerically.

Children care about what happens in real-life situations and are motivated to think hard. A striking example occurred when a mother gave a check for $5.75 to a second-grade teacher to buy a popsicle for each member of the class on her son's eighth birthday. Each popsicle cost 25 cents, and the teacher hesitated to give a problem like dividing 575 by 25, which is found in fourth-grade textbooks. The teacher nevertheless said to the class, 'I have to know *right now* whether this is enough money for everybody to get a popsicle, because either everybody will get one, or nobody will get any.' The solution the children invented was to count

the members of the class by saying, 'One-two-three-four (people); that's one dollar. One-two-three-four (additional people); that's two dollars. One-two-three-four (additional people); that's three dollars. . .'. They thus informed the teacher that there was exactly the amount necessary for everybody to get a popsicle.

Group games

Repetition is necessary for children to learn sums and products, but there is a vast difference between repetition through games and repetition through worksheets. An example of a game is a modification of 'Old Maid'[2] in which children try to make a total of ten with two cards. If the cards used in the game go up to 9, the possible combinations are: 9 + 1, 8 + 2, 7 + 3, etc. Games have traditionally been used as rewards for pupils who have finished their work, but we use them as a main staple.

The first advantage is that, in games, children are motivated from within to learn arithmetic. They often beg to play games but never beg to fill out worksheets. The great majority of them complete worksheets only because the teacher wants them done.

Another advantage of games is that children supervise each other, and feedback is immediate. For example, if somebody uses a 7 and a 4 to make ten in the preceding modification of 'Old Maid', another player is likely to object to this combination. When worksheets are used, by contrast, the teacher usually corrects them later and returns them the next day. By the next day, however, children cannot remember what they did the day before and do not care what happened yesterday!

The teacher's attitudes to worksheets are undesirable also because they foster children's dependence on adults. We want children to develop confidence in their own ability to figure things out and to make their own judgements. Having to depend on the teacher's omniscience hinders children's development of self-reliance and confidence.

Most educators, as well as the public, think that mathematics education is one thing, and that children's socio-moral development is quite another. Piaget (1973) pointed out, however, that the exchange of points of view is absolutely essential for children's intellectual development, as well as for their socio-moral development. This is because all children are egocentric and can think about things only from their own limited perspective. To become able to think about other relevant factors, children need to exchange ideas with others and to try to co-ordinate many viewpoints. This is a very complicated point. Suffice it to say here that children do not develop socially or morally by working alone sitting in their places completing worksheets. The reader interested in further details about children's socio-moral development are referred to Kamii (1982, 1985, 1989*a*, 1994). Many more games can also be found in these books.

Discussions of ways to solve problems

Textbooks usually present computational techniques first and then introduce word problems to get children to apply these techniques. We proceed differently and in the opposite sequence. We give word problems first, never tell children how to solve them, and encourage them to invent their own ways of getting the answer. In first grade, for example, we may ask how many children in the class could vote for the second alternative, if there are two choices and thirteen of them have already voted for the first alternative. In second grade, we may ask if there are enough crackers in two boxes for everybody to get three of them.

Note that these problems are closely related to children's daily living. We present these kinds of word problems first because numerical reasoning grows out of children's logico-mathematization of reality. Textbooks present computational techniques first because their authors assume that arithmetic is social (conventional) knowledge, or part of our cultural heritage that must be transmitted to the next generation.

Young children tend to solve most problems using addition. To solve the preceding problem about voting, for example, many count up from thirteen. To answer the question about crackers, second graders usually do 3 + 3 + 3 + 3 . . . or 22 + 22 + 22 if there are 22 children in the class. To say that the first problem is a subtraction problem, or that the second one is a multiplication (or division) problem is a completely arbitrary imposition. Discussions are sometimes about computational problems, especially when a procedure is new and difficult for the class. For example, children begin to tackle two-digit addition problems at the beginning of second grade. These are enormously difficult for many second graders, and the teacher writes one problem after another such as the following on the chalkboard and asks, 'What's a quick and easy way of solving this problem?'

FIGURE 1: Some two-digit addition problems

9	4	15	13	18
+5	7	+ 6	+13	+14
	5			
	2			
	5			
	+3			

The entire class can work together, or the teacher can work with small groups. The children raise their hands when they have an answer.

When most of the hands are up, the teacher calls on individual children and writes all the answers given by them. Being careful not to say that an answer is right or wrong, the teacher then asks for an explanation of each procedure invented by the children. For the first problem (9 + 5), for example, if a child says, 'I took 1 from the 5 to make 10,' the teacher crosses out the 9 and the 5 and writes '10' next to the 9. If the child then says, 'That made the 5 be 4,' the teacher

writes '4' below the 10. If the child concludes by saying, '10 and 4 is 14,' the teacher draws a line below the 4 and writes the answer, '14,' below this line as well as below the line in the original problem.

As the teacher thus interacts with the volunteer, he or she encourages the rest of the class to express agreement or disagreement and to speak up immediately if something does not make sense. The exchange of points of view is very important in a Piagetian, constructivist programme, and the teacher is careful not to reinforce right answers or to correct wrong ones. If the teacher said, 'That's right', all thinking would stop immediately. As long as the teacher does not say that an answer is correct or incorrect and, instead, encourages the children to agree or disagree among themselves, the class will continue to think and to debate until agreement is reached.

Many teachers ask, 'What should the teacher do if no one in the class gets the right answer?' Our reply is that, if this happened, the teacher would know that the problem was too hard for the class and would go on to something else. In the logico-mathematical realm, if children debate long enough, they will eventually get to the correct answer because absolutely nothing is arbitrary in logico-mathematical knowledge. For example, 18 plus 14 equals 32 in every culture because nothing is arbitrary in this relationship. The reader interested in more detail about this way of teaching is referred to Kamii (1989*a*, 1989*b*, 1990*a*, 1990*b*).

A conclusion reached on the basis of my research is that children can indeed invent their own computational procedures, and that their procedures go in the opposite direction to conventional algorithms. For example, in addition, subtraction and multiplication, children are now taught to proceed from right to left, that is, from the column of ones to those of tens, hundreds and so on. When they are free to do their own thinking, however, they invariably proceed from left to right. To do 38 + 16, for example, they typically do 30 + 10 = 40, 8 + 6 = 14, and 40 + 14 = 54. Figures 2 to 4 show the various ways children usually invent for addition, subtraction and multiplication.

FIGURE 2: Three invented procedures for solving 18 + 17

10 + 10 = 20	10 + 10 = 20	10 + 10 = 20
8 + 7 = 15	8 + 2 = another ten	7 + 7 = 14
20 + 10 = 30	20 + 10 = 30	14 + 1 = 15
30 + 5 = 35	30 + 5 = 35	20 + 10 = 30
		30 + 5 = 35

FIGURE 3: Three invented procedures for solving 53 – 24

50 – 20 = 30	50 – 20 = 30	50 – 20 = 30
3 – 4 = 1 less than 0	30 – 4 = 26	30 + 3 = 33
30 – 1 = 29	26 + 3 = 29	33 – 4 = 29

FIGURE 4: Two invented procedures for solving 125 x 4

4 x 100 = 400	4 x 100 = 400
4 x 20 = 80	4 x 25 = 100
4 x 5 = 20	400 + 100 = 500
400 + 80 + 20 = 500	

When a problem involves division, the law of the land suddenly changes, and the rule decrees that students work from left to right! If children are encouraged to do their own thinking, however, they proceed from right to left as can be seen in the division of 74 by 5. They typically do 5 + 5 + 5 + 5 + 5 + 5 + . . . , until the total comes close to 74. Note that when the second 5 is added, there is a shift to the left, from the ones column to the tens column.

Children usually count on their fingers saying, '5, 10, 15, 20 . . . ,' counting on five fingers. They then go on to say, 'If 5 fives is 25, 10 fives is 50. Four more fives is 20, and 50 + 20 = 70. So the answer is 14 fives, with a remainder of 4.' This method later becomes shortened to: 10 x 5 = 50, 4 x 5 = 20; so the answer is 14 with a remainder of 4.

EVALUATION OF THE RESULTS

To evaluate the results of constructivist teaching, I have compared the answers given by traditionally instructed children with the responses given by children who have never been taught algorithms. As can be seen in Kamii (1989*a*, 1989*b*, 1994), the children who did their own thinking consistently did better than those who received instruction in how to produce correct answers. I will give two examples of the kinds of questions asked in these evaluations and the kinds of responses given by each group of children. The traditionally instructed group, who were taught algorithms, will be referred to as the 'traditional' group. Those who had never been taught algorithms and invented their own solutions will be referred to as the 'constructivist' group.

If our goal is to get children to *think* numerically, our evaluation of the results must focus on their reasoning. This is why we evaluated children's thinking in individual interviews rather than examining only the correctness of their answers.

13 x 4

One of the problems given to third graders in individual interviews was 13 x 4 written vertically on a blank sheet. All the children in both the 'constructivist' and 'traditional' groups wrote the correct answer of 52 as can be seen in Table 1. (In the 'constructivist' class, there were only thirteen children who had never been taught any algorithms in school in grades 1 to 3.)

TABLE 1: Percentages in the 'constructivist' and 'traditional' groups explaining how they got the answer to 13 x 4

	'Traditional' group (*n*=39)	'Constructivist' group (*n*=13)	Difference	Signif. (2-tailed)
Correct answer (52)	100	100	0	
Use of algorithm	97	0	97	.001
Adequate explanation of all the steps	5	92	87	.001
Interpreting the '1' in '13' as a 1	87	0	87	.001

When the child finished writing the answer, the interviewer brought out a bagful of chips and asked, 'If we make four piles of 13 and 13 and 13 and 13 chips (indicating four different locations on the table in front of the child), will we have what this problem says?' All the children in both groups replied in the affirmative, and the interviewer and the child together made four piles of chips, each containing thirteen.

The adult then asked, 'If we pushed all these chips together, how many would we have?' All the children in both groups replied, correctly, '52'.

The interviewer went on to ask the child to explain with the chips (which were still in four piles of thirteen each) 'how all this works (pointing to what the child had written).' The adult purposely did not ask the child to show 'what you meant when you wrote this' because she knew that most of the children did not understand the 'why' of the rules they were using.

As can be seen in Table 1, 97% of the 'traditional' group used the conventional algorithm, but none of the 'constructivist' group did. The significant finding was that only 5% of the 'traditional' group adequately explained all the steps they had written, while 92% of the 'constructivist' group explained what they had written. The most frequent reason for the 'traditional' group's inadequate explanation was their poor knowledge of place value. As can be seen in Table 1, 87% of the 'traditional' group indicated only four chips (rather than forty) to demonstrate what '4 x 1' meant in computing 13 x 4.

When a child showed only four chips to explain the '4 x 1' part of 13 x 4, the interviewer remarked, 'You used all these [*pointing to the chips the child had used to explain 4 x 3 and 4 x 1*] to explain how "all this" works [*pointing to the child's writing*]. But you didn't use any of these [*pointing to the unused chips*]. Were you supposed to use all of them, or were you not supposed to?' The adult thus asked questions that might prompt a better explanation if the child did not adequately explain the written procedure on his or her own. These questions, however, did not help 87% of the children in 'traditional' group, who replied that it was not necessary to use all the chips to explain the algorithm.

6 + 53 + 185

Another problem given to third graders in individual interviews was 6 + 53 + 185 written horizontally. The children were asked to work the problem in their heads without pencil and paper. As can be seen in the upper part of Table 2, 50% of the 'constructivist' group produced the correct answer, while 32% and 20%, respectively, of the 'traditional' classes did. A more significant difference was found in the wrong answers the groups produced.

TABLE 2: Answers to 6 + 53 + 185 provided by three classes of third graders

'Traditional' group (*n*=19)	'Traditional' group (*n*=20)	'Constructivist' group (*n*=10)
Percentage with correct answer		
32	20	50
Incorrect answers		
	800 + 38	
838	800	
768	444	
533	344	284
....	....	....
246	243	245
235	239	243
234	238	238
....	234	....
213	204	221
194	202	
194	190	
74	187	
29	144	
-*	139	
-*	-*	
	-*	

* The child declined to try to work the problem.

All the incorrect answers given by two 'traditional' classes and one 'constructivist' group are listed in Table 2. The dotted lines in this table were drawn to highlight the wrong answers that were within a reasonable range. It can be seen in this table that the incorrect answers were much more reasonable in the 'constructivist' group than in the two 'traditional' classes. In other words, the 'constructivist' group more frequently produced the correct answer, and if a child in this group got an incorrect answer, the wrong answer was more likely to be reasonably wrong.

Many more data can be found in Kamii (1989*a*, 1989*b*, 1994) about word problems, many other kinds of computational problems, and teachers' frustration in the classroom as they tried to undo the harm caused by algorithms.

The harmful effects of algorithms

An unexpected finding of the evaluation research described above is the conclusion that the teaching of algorithms is harmful to children's development of numerical reasoning. Two reasons can be given to explain this conclusion.

First, children have to give up their own thinking to use the rules of 'carrying,' 'borrowing,' and so forth. As stated earlier, these rules make children go from right to left in addition, subtraction and multiplication, that is, from the column of ones to those of tens, hundreds, and so on. When children are free to do their own thinking, however, they invariably proceed in the opposite direction. Because a compromise is not possible between going from left to right and going from right to left, children have to give up their own thinking to obey their teachers.

The second reason for saying that algorithms are harmful is that these rules 'unteach' place value and prevent children from developing number sense. When children are taught algorithms and solve problems such as 5 x 234, we can hear them say, '5 times 4 is 20, put down the 0, and carry the 2. 5 times 3 is 15, plus 2 is 17, put down the 7, and carry the 1, ...' Treating every digit as ones is efficient for adults, who already know that the 2 in 234 is 200. For primary-age children, who have a tendency to think that the 2 in 234 means *two,* however, algorithms reinforce their weakness. By contrast, children who do their own thinking say, 'Five times 200 is 1,000, etc.', thereby strengthening their knowledge of place value.

In both examples given in the discussion of evaluation, we saw the difficulty of place value in the 'traditional' group. These children thought that the '4 x 1' part of 4 x 13 meant 'four times one'. They also got an answer in the 700s by adding the 6 and the 1 in 6 + 53 + 185. If they carried 1 from the column to the right, their answer came out in the 800s. Table 2 supports the assertion that children who are taught algorithms have poor number sense. Poor knowledge of place value causes poor number sense. As can be seen in Kamii (1994), children's wrong answers become even more outlandish in fourth grade! Answers such as 1,215, and those in the 700s and 800s, become more frequent after one more year of algorithms.

Conclusion

For centuries, education has been a folk art based on tradition and opinions called philosophies. With the advent of associationism and behaviourism, however, education began to enter a scientific era. Now that we have Piaget's constructivism—a more adequate scientific theory—it is time to change the way arithmetic is taught in elementary schools.

I hope educators and researchers in other countries will also experiment in classrooms to test the hypothesis that I tested. Because logico-mathematical knowledge is universal and the same in all cultures, I expect the same kinds of results in other countries.

Constructivist teaching is much harder than the teaching of algorithms and correcting of worksheets. However, in spite of this difficulty, an increasing number of teachers in the United States are becoming convinced of the truth of Piaget's constructivism. When these teachers encounter children's originality and intelligence that they have not observed before, they are truly overjoyed. Once they have seen the possibility that exists in each child, these teachers do not think of going back to their old ways of teaching.

Notes

1. I am grateful to Janice K. Ewing for critically reading a draft of this paper and making helpful suggestions.
2. 'Old Maid' is a simple card game in which the player holding a designated card at the end of the game is an 'old maid'.

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THE EXPANSION AND INFLUENCE OF PIAGETIAN THEORY ON EDUCATION IN JAPAN

Takehisa Takizawa

No one will deny that modern Japanese psychology and education have been profoundly influenced by Piaget and his theory. Three stages can be distinguished in the spread of that influence: introduction, consolidation and expansion.

Introduction (1931–70)

It was only after 1960 that Japanese psychologists began to take an interest in Piaget's theory in all its aspects. Until then, they had seen him solely as a child psychologist, the author of an interpretation of children's thinking based on the concept of egocentrism.

Admittedly, the name of the Swiss scientist was known very early on in Japan: as long ago as 1931, Hatano collected five of his early articles in a publication entitled *Jidô Shinrigaku* [The psychology of the child]. His theory of egocentrism was subsequently to provide inspiration for numerous studies of children's thought processes.

But it was only in 1954 that the first Japanese translations of his writings were published, and then only of his early work. Then, in 1960, *The psychology of intelligence* gave a comprehensive view of his theory to the Japanese in their own language.

Takehisa Takizawa (Japan)
Obtained his M.Sc. in education at the University of Tokyo, where he was an assistant lecturer from 1958 to 1961. He was a lecturer at the University of Niigata, starting in 1961, and took up an appointment in 1967 as assistant professor at the University of Electro-Communication of Tokyo, where he is now professor of psychology. Since 1992, he has been President of the Franco-Japanese Association of Educational Sciences. He has published a number of works, including *The development of the theory of Jean Piaget* (1992).

The early 1960s witnessed a marked vogue for Piagetian theory as a result of the modernization movement in Japanese education, the originator of which, J.S. Bruner, established a doctrine which owes much to the Swiss psychologist.

Thereafter, commentaries and research multiplied on the work of the middle and final periods of Piaget's career, with translations coming in quick succession. More particularly, the publication in Japanese of *The child's conception of number* in 1962 and *The child's construction of quantities* in 1965 were of outstanding significance, as they marked the entry of these subjects into the field of inquiry of researchers in genetic psychology.

Japanese psychology, being a faithful disciple of American psychology, followed in its footsteps in its rediscovery of Piaget and emulated the research on his theory that was beginning in the United States. To everyone's surprise, the numerous experiments carried out, whether they concerned the concepts of number, quantity, space and time or the operational development of the inclusion of classes and relationships, produced results which often confirmed Piaget's predictions.

At the same time, genetic psychologists had an urgent task to perform: to carry out fundamental research on the early development of intellectual abilities, which was one of the objectives of the educational modernization movement. Extrapolating from their work on the development of logical and mathematical concepts in children, they tried to find ways of encouraging that development through the learning of these concepts. They were often to use empirical educational techniques for the purpose.

The method consisted in using preliminary tests to select children who had not yet reached a certain threshold of knowledge and subjecting them to a battery of programmed exercises (external consolidation, cognitive conflict, etc.). The results of exercises performed during final tests or transfer tests were evaluated in order to gauge the effectiveness of the learning process. Many researchers adopted this approach in order to determine under what conditions and using what educational programmes it was possible to achieve a measure of success and to raise the cognitive level of the child. Fujinaga et al. (1963) thus demonstrated through experiments on the learning of the concept of number in early childhood that it was preferable to begin with exercises in which pairs of numbers were compared, rather than to start immediately by teaching tables of operations or arithmetic.

On the basis of this evaluation process, these learning experiments can no doubt be used to identify the factors which govern cognitive development. On the other hand, the brevity of the exercises concerned makes it necessary to restrict the field of research to the development of concepts in a limited number of cognitive domains. It is thus difficult to transpose the results of the experiments to the actual classroom. The application of Piaget's theory in the classroom calls for methods to be worked out which suit each specific educational content.

During that same period, learning methods and materials directly derived from Piaget's theories as well as experiments described by him were set up *in situ.*

The experimental tools and illustrated cards he used were introduced unchanged into the classroom, in the hope that their use by children would speed up the development of their cognitive abilities. A directive approach was chosen. The children were not free to handle the educational materials as they liked; they performed operations which were mainly determined by the teacher, and then answered questions, with the degree of accuracy of their replies being made clear to them in order to consolidate their acquisition of the right concepts.

This approach was in fact at variance with Piaget's true intentions which, on the contrary, were to focus on the child's active participation and spontaneity. The educational world of the day had only superficial knowledge of that theory and, on the whole, placed upon it a teacher-oriented interpretation which showed signs of the influence of the educational modernization movement.

For example, Yokochi (1964), to whom we owe this kind of method, contended that intellectual education is impossible for a child whose everyday universe is dominated by complexity and contingency. In order to accelerate cognitive development, it was above all necessary to define topics which were as simplified as possible and to teach them in a systematic and rational way by bringing out their structural qualities.

Piaget's theory, which was accused of giving undue emphasis to intellectuality in his doctrine on the early development of abilities, then became the butt of harsh criticism from believers in a traditional education based on the child's sensitivity and freedom. In fact, contemporary educators imposed a severely reductionist interpretation on it, and those who grasped its theoretical complexity were few. Most of them retained only the idea of level of cognitive development and thought that in order to develop children's intellectual capacities, it was crucial to stimulate them intellectually. In particular, a method of teaching was established which emphasized the use of language to express logical thought and which relied on the verbal formulation of both the teacher's explanations and the child's replies to accelerate development.

Consolidation (1970–80)

The visit by Piaget and Inhelder to Japan in June 1970 afforded an opportunity to clarify his theory. The paper which they read to the Kyoto world conference on education and childhood enabled Japanese specialists to get to grips with the original version of Piaget's ideas and to overhaul their views on the education of children.

The Japanese Society for the Education of Children (Nihon Yônen Kyôiku Kai), which organized the conference, had long been striving to establish an educational method based on Piaget's theory, encouraged by contacts between national researchers and the work to establish and evaluate curricula, methods and materials which involved practitioners and researchers. Until then, its approach had laid primary emphasis on the teaching of Piagetian themes and had given an excessively directive role to the teacher.

In this regard, Piaget's stay in Japan marked a turning-point. Thereafter, the educational process was to give priority to the wealth of the child's spontaneous experiments in thought, judgement and expression. It would take a two-fold axiom as its premise: that children develop through their spontaneous actions; and that children develop by becoming aware of their mistakes and correcting them themselves.

Teaching materials were now designed not to accelerate the child's development but to consolidate his or her groundwork. Cognitive development is based *inter alia* on a whole range of operational experiences defined by Piaget's theory, the idea being that the concepts of number, quantity, space and time do not originate separately from one another. Children should be helped to acquire the ability to observe things and to appreciate their relationship to other things and to themselves, by being made to think while they carry out practical operations (removing, adding, shifting) on drawings which represent concrete objects (Matsui, 1976).

In the field of genetic psychology, rather than the wholesale adoption of Piagetian precepts, in-depth research was conducted which revealed a distinctive characteristic in the development of Japanese children's thinking. That characteristic was thought to be partly due to a question of language. For example, as regards the development of the concept of number, it was discovered early on that the complexity of the numerical terms in Japanese hindered the counting activities of children (Kawaguchi et al., 1962). It was also observed that the nuances of the vocabulary used for the comparative influenced their reactions somewhat. Thus, on the subject of conservation, Takeda (1977) observed that questions requiring the replies *mijikai* (short) or *karui* (light) received a smaller number of correct answers than questions requiring the replies *nagai* (long) or *omoi* (heavy). Mori (1976) compared the acquisition of the concepts of speed and time between Japanese and Thai children. Japanese has only one word, *hayai,* for both 'soon' and 'fast', while the words *nagai* (long) and *mijikai* (short) refer to both duration and distance. As a result, Thai children show better results than Japanese children in the Piaget experiment which entails judging the speed and duration of movement of two moving objects.

These studies not only proved that the role of language in the development of thought should be reconsidered, but, above all, cast doubt on the universality of Piaget's theory and militated in favour of the establishment of a theory of cognitive development which would be really appropriate for Japanese children.

At the same time, the educational world was no longer content with merely applying Piaget's methods and principles, but strove to return to the spirit underlying his theory and to emphasize its more specific aspects. The Suidô (water supply) method of teaching mathematics played a pioneering role in this shift of emphasis. Its originator, Hiraku Tôyama, became interested very early in Piaget's theory, circulating the works of the Swiss researcher to Japanese teachers and participating in the translation of *The child's conception of number* and *The child's construction of quantities.* He realized that, just as Piaget had stated that the

thinking of children was not an imperfect version of adult thinking but possessed its own structure, so elementary mathematics was not an incomplete version of advanced mathematics but, equally, had its own structure.

Piaget's theory would remain a constant reservoir of ideas for the theory of mathematics teaching, inasmuch as its first principle is that the psychology of the operations which constitute the basic thought processes are derived from mathematical structures. Thus, when the teaching of the new mathematics began, Piaget immediately cautioned against the general tendency to employ traditional teaching methods without paying attention to the relations existing between mathematical structures and the spontaneous operational structures of the child. Tôyama's team responded to Piaget's warning by introducing the Suidô method (1960).

This is a method of mathematics teaching based on a metaphor: one need only collect (standard combination stage) mountain water (elementary processes) in a reservoir for the water eventually to overflow and supply all the connected outlets (specialized combination stage). In other words, one need only teach a sufficient range of concrete problems which embody general principles, to be able subsequently to infer from them the solution to all other problems. The method is modelled on the work of Piaget, who was able to predict, by inference, the reactions of children in a wide variety of situations on the basis of their overall thought structures.

But Piaget emphasizes the internal operational structures which are spontaneously used by the child, whereas the Suidô method adopts a signposted approach which is based not on those structures but on the numerical structures newly acquired by the child. Whereas traditional mathematics teaching, which is based on mental arithmetic, produces generations of children who do not understand the structure of Arabic numerals and who can count mechanically but cannot understand the meaning of numbers, the Suidô method begins with the initial step of understanding the structure of numbers. It emphasizes the concept of number based on quantity.

The child is made to approach the structure of numbers by handling square tiles. Based on Cuisenaire's teaching material, tiles are a practical means of ensuring that the structure of Arabic numerals is understood. They can be arranged edge to edge, both vertically and horizontally. Units of ten are shown by the formation of complete columns of ten tiles, and units of 100 by squares of 100 tiles (ten columns). This characteristic of tiles gives the child a concrete picture of the meaning of the calculations.

The Suidô method also steers learning towards written, not mental, arithmetic by establishing a supervised procedure which classifies the exercises according to certain principles, ranging in order from the more general to the more specific.

To take the case of addition, the teaching method previously used was of a kind which brought together ordinary combinations and special combinations of numbers: 2 + 2, 20 + 20, 20 + 2, 22 + 2, 9 + 9. But the Suidô method begins by teaching additions of one-digit numbers such as 2 + 1, 0 + 3 (elementary stage),

then continues by linking them to standard additions such as 22 + 22 (standard combination stage) and ends with non-standard additions such as 22 + 20 and 22 + 2 (specialized combination stage).

This unified system of supervised learning has obvious points of similarity with Piaget's theory. The Suidô method is thought to have been presented for the first time in 1958, but it was only in the 1960s and 1970s that it created a considerable stir in the world of education.

Expansion (from 1980 to the present)

After Piaget's death, his theory led off in new directions in Japan as a result of work aimed at reviewing cognitive development in the context of the mental activity of the child in his or her everyday life. According to Piaget's doctrine of interactions, cognitive activity cannot be explained solely in terms of an internal mechanism in the child. The questions it raises should be reformulated differently: what relationships do children have with their immediate environment? What do they derive from it to construct their own thinking?

Children derive a great deal of knowledge from their everyday experience which is destined to be put to use, by various mechanisms, in their intellectual processes. They probably use this knowledge to understand the events, even unfamiliar ones, which happen in their lives. This also applies to their animistic thought, about which there seems to be agreement that it, too, is not spontaneous and natural but reflects a deliberate act of drawing analogies between familiar human representations and unknown situations.

Thus, Inagaki and Hatano (1990, 1991) claim that children perceive the existence and fate of animals and plants through analogies based on their knowledge of human existence, but also that this process is not indiscriminate. For example, if they are dealing with animals or plants which are sufficiently familiar for them to have some knowledge of their characteristics, the children can then apply that knowledge. However, both the everyday world of children and the scope of their knowledge are limited, and they often have no option but to employ anthropomorphic analogies.

Whenever children have new and repeated experiences in their everyday environment, they will be increasingly attentive and will try to work out the meaning of such experiences and events around them, all of which should add to their body of cognitive knowledge and so stimulate the thinking which draws upon it. Such knowledge is admittedly naïve, and even inaccurate, but Ogara (1990) has demonstrated that the acquisition of scientific knowledge takes place more smoothly, following appropriate readjustment, through the analysis of naïve or inaccurate knowledge acquired in everyday life than through the exclusive learning of facts in school.

Reflecting this research, Japanese teaching nowadays also emphasizes the interdependence between concrete experience and thought, and makes considerable provision in the classroom for empirical learning. Thanks especially to

lessons on life and the environment, a new subject in the first and second years of primary school, the children's field of learning is expanded to include their entire sphere of existence, and their bodies become versatile tools for learning through the experience of social and natural events taking place in their immediate environment. Their learning progresses through spontaneous actions in which they identify with the object—looking, comparing, creating, exploring, lifting, playing—and in which the teacher is no longer the unilateral guide. The emphasis is on the relationship between the child and the environment rather than objective understanding of society and nature.

Children are interested in and gain awareness of the significance of the environment for them and, at the same time, improve the perception they have of themselves and of their existence by repeating actions in which they are the principal performers. That is precisely the purpose of the lessons on life and the environment: to develop that perception so as to prepare the child to cope as effectively as possible with what must be learned in the years ahead.

Such lessons constitute an essential underpinning of Piaget's theory, and may be regarded as one of the most successful educational applications of that theory in Japan.

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PIAGET, EDUCATION AND INTERCULTURAL PERSPECTIVES

Mohamed Lahlou

Piaget's name is more often associated with genetic epistemology and universals than with teaching methods and interculturality, but who more than he has applied psychology to education, and universality to an intercultural approach to human thought processes?

The academic ground he covered in his lifetime certainly justifies his standing as an educationist, even if all the specialists agree that for him education was only a secondary interest. A naturalist, Piaget's passion for biology provided him with his basic model of the subject coming to grips with the world, as he studied the genesis of knowledge in the context of development of individual thought. It was his work on genetic epistemology that was to be most fruitfully applied to education, emphasizing the role of action in the development of the child's intelligence and knowledge.

Piaget's early interest in active learning led him, at a decisive stage, to collaborate with Claparède and then to become an authority in his own right, at the head of leading teaching and education institutions. Piaget was thus a genetic epistemologist who worked on the genesis and transmission of knowledge.

The name of the researcher who, on the basis of observation of children in Geneva, constructed models which he posited as universals, is not linked as readily, however, to interculturality. His work has prompted many comparative studies, which although not qualified as 'intercultural' are basically intercultural approaches and comparisons. It must also be said that the application of his theo-

Mohamed Lahlou (Algeria)
Professor of intercultural psychology at the Université Lumière Lyon-II (France). Professor of psychology at the University of Algiers for several years. Mr Lahlou holds a Ph.D. in psychology and a Ph.D. in humanistic studies. His published works deal with the cultural factors of individual and collective development. As research director, he has also supervised many theses in psychology. He is the founder of the *Revue algérienne de psychologie* and is now working mainly on the problems of interculturality and social representation.

ries to education has, by virtue of the diversity of the population groups to whom education programmes cater, involved setting them against the background of the cultural realities—the demands—of children's immediate environments.

It is this connection between Piagetian teaching methods and intercultural perspectives which makes Piaget's work all the more relevant in countries like those in Africa that must build ideologically unified societies on a pluralist basis and set up education systems that encourage individual and group fulfilment while strengthening the universal processes of rational thought, which are essential for the development of scientific ideas.

Educational needs and educational requirements

The term 'underdevelopment', used so frequently immediately after decolonization, has discretely disappeared from today's vocabulary, as if to draw a veil, firstly, over the failure of the development programmes undertaken in formerly colonized countries and, secondly, over the growing imbalances between North and South. This imbalance, which is already substantial in economic terms, is quite striking in the socio-cultural field in which established institutional systems confine their educational objectives to 'the technical acquisition of information' with no reference to the cultural preconstructs that shape the child's inner personality.

EMERGENCY LITERACY CAMPAIGNS AND EDUCATIONAL CENTRALISM

In addition to its very real demographic and economic problems, the African continent has failed, despite the large budgets frequently allocated to education, to take off in education and culture because its educational curricula did not build bridges between the universality of thought structures and the cultural specificity of certain educational contents.

Furthermore, as illiteracy rates were very high when its nations became independent, Africa focused its attention on literacy rather than on educational objectives geared to the quickening build-up of knowledge in the developed world, as if African schools had perforce to go through the stages of development traversed by the schools of the colonial powers. States and government authorities, weighed down by problems they considered inevitable, resigned themselves to going from one educational emergency to another. The crushing weight of colonization and the high cost of decolonization caused acute problems for fledgling nations, with no intellectual or scientific élites, that were still struggling to achieve food self-sufficiency.

The structure that African countries inherited from colonial and pre-colonial times was established by reference to the structure of the European colonial nations and based primarily on a centralizing approach to authority and institutions, which was applied to both the State and State schooling. Educational and

cultural life was thus managed in a standardized, authoritarian way, and educational institutions forgot that the child should be the focus of all education curricula and that he or she brought to school an original culture that inevitably played a part in his or her acculturation. Faced with a plurality of national cultures, African schools have in general disregarded their pupils' cultures of origin and, for the sake of modernity—indispensable for development—opted for a culture of education inspired by the cultural model of the industrialized West. They have selected from all the information that it conveys whatever was considered to be a 'free culture', in the conviction that they were acquiring universal knowledge. The trouble was that that knowledge was divorced from the context which would enable children to assimilate it.

LANGUAGES OF INSTRUCTION

The ideological question also left its mark on the choice of languages of instruction. True, Africa had a variegated language heritage that was made even more unwieldy by the absence of linguistic, psycholinguistic, psycho-educational and sociolinguistic analysis of all the languages concerned. Education officials responded by opting very rapidly for a 'unifying' language system based either on the language of the former colonizer or on the language of a dominant 'component', and thus marginalizing the other languages and their speakers. The acquisition of a new language has not been easy; the proficiency levels of most schoolchildren in their new language have remained far below those required for the transmission of the educational and cultural content needed to build up an internationally competitive body of knowledge. In addition, the capacity of mother tongues to replace the language of instruction for the formal teaching of knowledge and skills and their incorporation into both the individual and collective cultural heritage has been reduced to a minimum. And so, while failing to develop an adequate and general level of mastery of the language of instruction, school systems have been responsible for the decline of native languages, and of their ability to renew and strengthen their own cultural productions and to offer a link to the cultural heritage of the world as a whole.

African governments have justified this strategy by pointing to the approach of the former colonial powers to their own regional languages and by arguing that it made for greater uniformity in education systems and educational content. Even when an effort has been made to acknowledge mother tongues, it has been confined to literacy programmes rather than aiming to promote those languages and to include them in a comprehensive programme designed to make each mother tongue a language of instruction. When they have not been banned, African languages have been reduced, on the pretext of authenticity and tradition, to a 'heritage' preserved in aspic.

In addition to the problem of mother tongues, African schools found themselves at the centre of an ideological confrontation between the system of thought conveyed by the languages of the former colonial powers and that conveyed by

national languages. Torn between their culture of origin and the school culture, pupils are faced with two often contradictory systems of reference, each conveyed by a language with its own history in terms of culture and civilization, and each confined in a completely watertight educational objective. On the one hand they have the language to which is assigned the role of managing tradition and extolling the past, and on the other the one whose task is to convey modernity and future-oriented projections; on the one hand the language of religion and on the other the language of secular concerns. They also have to teach the principles of morality on the one hand and, on the other, the one used to assume the principles of responsibility. The child is therefore torn between different systems of thought with conflicting points of reference, and education increasingly takes the form of conditioning based on *ex cathedra* discourse conveying concepts developed with no reference to the pupil's own activity.

Piaget's educational methods and the intercultural approach are of invaluable assistance here in that they stress the need for pupil-centred, active methods and an educational approach that takes in the contexts in which the educational process takes place and the ways in which these concepts are likely to develop.

Contributions of Piaget's educational psychology

Piaget's work on education, when read without the restrictions often imposed upon it in the past, can open up new avenues of thought for all educationists. It provides answers to educational concerns by linking the establishment of cognitive structures and the definition of educational objectives, by proposing that all educational activity should be based on the pupil's own activity and on environment-shaping factors that encourage cognitive development.

STAGES OF DEVELOPMENT AND CATEGORIES OF KNOWLEDGE

The educational interest of Piaget's work lies firstly in the procedure he used to make educationists aware of the child's thought processes and the conditions under which intellectual structures are established at different ages. By proceeding on the basis of a fixed order of development stages and taking into account contextual differences, educationists would be offering children the best possible framework for acquiring knowledge. They would also find in Piagetian concepts of number, space, time, causality, series, combination, etc., an indispensable intellectual basis for their teaching material.

Piaget's first-hand contact with children makes him a focus of interest for educators trying to come to grips with the genesis and development of intelligence; similarly, his use of the clinical approach is a model for pupil/teacher relations based on understanding of the child's behaviour. Experiments with the standardization of tests of reasoning carried out in Binet's laboratory also constitute a mine of information for educationists; the children's successes and failures in carrying

them out made clear the form taken by the cognitive processes used, the genesis of basic logical structures and the development of categories of thought.

ROLE OF SOCIAL INTERACTIONS

In addition to factors of neurological maturation (a requisite for the emergence of certain forms of behaviour), equilibration (which takes operations towards broader and higher forms of equilibrium) and exercise (through situation-based action), Piaget stressed, for educationists, the crucial role of social interaction and transmission as factors of development. This factor of social change entails a need for the family and the social group to participate actively, through the various cultures and subcultures, in the organization of school learning and the adaptation of content to form.

Some authors have been so struck by the rigour with which Piaget defined the stages of development that they have underplayed the importance given in his work to the role of social factors. These factors involve the interactions among the individual pupil, the teacher and other learners, and the interactions between pupils and their social environment as the stimulus and anchorage of formal learning. Hence the importance for education systems of encouraging children to discover, for themselves, in their own environment, the bases for logical thought, and of formulating in terms of logical principles the relations that their own environment establishes between events and objects.

RELATIONS BETWEEN PUPIL, SCHOOL AND IMMEDIATE ENVIRONMENT

In Piaget's opinion, intelligence is a means of adaptation which forms part of the individual's overall development and is based on all the adaptation processes that make it possible for an organism to reach a state of equilibrium within its environment. This concept of adaptation involves two processes: one encourages the absorption of environmental elements by the individual, the other concerns the transformation of the environment. A two-way process of assimilation and accommodation is established between the environment and the individual, and, as he or she develops, a balance is established through the absorption of environmental elements and a modification of thought structures. This proposition by Piaget is a key to restoring natural relationships between the pupil and the school and between the school and the pupil's immediate environment, in that it includes these three elements in the education process in a system of reciprocal participation and interdependent meanings. Hence too, the need to create, or to use as a basis, concrete conditions which will encourage a logical understanding of situations, even if that requires concentrating exclusively on pupils' preferred mode of cognitive operation before extending knowledge thus acquired to other contents. A structured environment capable of reinforcing skills in observation, verbal transmission and functional activity will stimulate the child's cognitive development.

ACTIVE METHODS

Piaget resolutely called for education to be based on active methods and took the functionalist view which establishes a basic link between the individual's activity on the one hand, and needs and aims on the other; it is a view which holds that action is the source of all knowledge. He thus rejected traditionalist methods which attribute a passive role to the pupil from the outset by presenting concepts in their most elaborate form. In his opinion, concepts should be built up gradually through the child's own activity, as the child takes an active part in his or her own development by assimilating different situations and integrating them into organized thought structures.

On the basis of his work on perception, mental images and intellectual operations, Piaget also rejected intuitive methods in which all knowledge is derived from sensorial experiments with objects, considering that it was only through the action exerted on them by pupils that knowledge of their properties would be built up. In sum, knowledge is derived from intervention in the real world. Piaget did not believe that the so-called programmed teaching methods could both make possible the acquisition of knowledge and develop the pupil's initiative by reinforcing his procedure based on trial and error and the postulation and testing of hypotheses.

Opposing these methods therefore, Piaget considered that active methods were the only ones that could enhance the child's capacity for adaptation, by incorporating assimilation-accommodation processes. Active teaching methods are based on the theory of interaction and constructivism, which holds that all intelligent acts involve both understanding a situation and devising a solution on the basis of that understanding. Educational activity must bring into play all the processes that determine the child's operational level: maturation, experiments with objects, social interactions and equilibration processes. Specifically, it must involve the individual directly by encouraging him or her to take direct action and by arousing curiosity which will lead to the discovery of solutions.

These, in our opinion, are the essentials of Piaget's methods, to be welcomed by schools that are seeking coherent models. But they will not be fully effective unless consideration of the child's environment takes into account the multiplicity of meanings and the coexistence of different cultures.

Education and interculturality

Schools—both in States with a long historical past and in younger States—have suffered from ethnocentrism and the uniformization of their cultural referents on the model of the ideological referents of those in power. African school systems, in particular, took over the centralizing approach of the colonial powers, who either ignored all education for 'autochthonous' populations or took 'ethnic' factors into account spasmodically, for administrative purposes, by introducing the teaching of 'indigenous' languages.

The intercultural approach did not begin to play a role in reflection on the organization of education systems until the theoretical principles that led to the birth of cultural anthropology had been developed. Building bridges between the social and psychological spheres, and between the individual and the group, this discipline declared its interest in the study of cultural phenomena, while teaching was seen as having a part to play not just in transmitting human cultures but in bringing them together.

ROLE OF INTERCULTURAL EDUCATION

While meetings between different cultures are nothing new, the multicultural character of educational activity has been recognized only very recently, and then only in exceptional cases, by institutions.

Multiculturalism, a reality in all societies, including those of modern times, encourages education systems to break with cultural ethnocentrism, opting rather for cultural democratization which makes it possible to bring educational and cultural objectives more into line with learners' needs. Multiculturalism, combining with the universality of thought structures, offers a new opportunity to the Piagetian education movement, which stresses the need for the environment to play a part in the acquisition of knowledge. The system of meanings specific to each group of individuals will indeed encourage the emergence of rules and standards anchored in human nature. Moreover, the dynamism of a culture depends on interaction between individuals and their community.

The way in which an individual relates to his or her cultural context and the way that context in turn relates with those around it make interculturality a combination of psychological, relational, institutional and other processes. These processes make it possible, through reciprocal exchanges, to study phenomena in a variety of practical contexts. The effectiveness of the intercultural education approach stems from this factual reality and from the interest that a pupil shows in environmental stimuli and in support from the group to which he or she belongs.

In addition to inputs from the comparative study of various cultural groups, intercultural education is based on the existence of individuals and groups who experience changes in cultural contexts or who are confronted with different cultural worlds (case of cultural minorities and migrant pupils).

THE SCHOOL AND PLURALIST INTEGRATION

The intercultural approach also underscores the basic role of the notion of adaptation through the two Piagetian processes of assimilation and accommodation. As the pupil is confronted with several systems of references, he or she is the focus of a dynamic equilibrium between enculturation and acculturation. This equilibrium must be borne in mind by the educationist in curriculum design and in the application of teaching methods.

Enculturation—taken to be all the processes through which the child absorbs the culture of his or her environment while being accepted by it—places the pupil in a learning situation marked by *reciprocal assimilation of the child and the environment.* Acculturation, for its part, responds to the needs of both educators and learners when direct and continuing contact occurs *among individuals with different cultures;* it expresses participatory activity during which balances are constantly being struck and readjusted.

Interculturality also makes it possible to emphasize the role of experience as a source of knowledge and the significance of cultural differences as acquired and transmitted habits. These habits are both acquired and transmitted in interactive relations in which all the operations available are involved in exchanges. This is a genuinely pluralist process of integration in that the pupil's culture is the outcome of co-operation in which all the cultures represented in his or her environment come together in a coherent whole within which the individual and the group retain their freedom and interdependence.

Intercultural education therefore enables education systems to make the various cultures with which the child is in contact play their part. It incites these cultures to join forces by introducing standards and common referents based on the recognition of differences in meaning and the originality of the different approaches.

As a response to the difficulties experienced by education institutions when faced with a range of different situations, the intercultural approach makes for a cohesive, participatory school and offers an opportunity for discourse based on identity that encourages knowledge of other identities and recognizes the multiculturality of the pupils' environment. This approach also leads to discovery of operations involving reciprocity, complementarity, seriation and interrelations. In sum, it is an approach to human relations based on mathematical logic which offers the benefit, on the affective level, of contact with and recognition of others.

USE OF THE MOTHER TONGUE

The intercultural approach permits a better grasp of language problems in countries where the only response to linguistic diversity has been an ideological one based on the principle of domination or exclusion, the justification given for this response being the alleged need for a means of expressing thought that matches those of surrounding ideologies or a rank-ordering of languages and cultures implying that certain languages and cultures are incapable of playing a part in the development of thought and the formation of knowledge.

By stressing that language enables the child to reproduce a specific model provided by the environment, the concept of interculturality is at one with Piagetian psycholinguistics which demonstrates, firstly, that the development of language is based on the acquisition of certain universal structures and, secondly, that to master certain structures of language, the individual must develop specific approaches. Hence the need to increase, through the acquisition of basic struc-

tures, recourse to the child's natural language as a readily available language register.

The notion of interactionism, common to the Piagetian approach and to the intercultural approach, highlights the ongoing dialectics between the individual and his or her environment. That of constructivism also links the two approaches together by showing that knowledge is developed through a set of choices and exchanges with the environment during the different stages of development. Acknowledgement of the linguistic plurality of learners will reveal the link between cognitive patterns of behaviour and language behaviour patterns, particularly in the description of objects and situations. If the terms used are borrowed from vocabularies that are in any way differentiated, they make it easier to understand how children answer questions and that those answers are not systematically considered incorrect. In short, the child's language is the medium used both to acquire knowledge and to express his or her representation of objects and situations. This is an essential principle for remaining in contact with reality and with the world of representations inherent in each human community.

The Piagetian education method, born of rational knowledge of the mechanisms of individual thought, still constitutes an important referent for attempts to meet the needs of education systems, particularly those that are grappling with the definition of objectives on the one hand, on the other, with children's need to live their own culture, and, finally, the needs of the cultures that participate in both institutions and knowledge. Piaget's approach is at one with the intercultural approach in its view of the understanding of human thought and the adaptation of educational principles to the boundless riches of human experience.

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THE ACQUISITION OF CULTURAL OBJECTS: THE CASE OF WRITTEN LANGUAGE

Emilia Ferreiro

Introduction

The purpose of this chapter is to show that, in spite of the fact that social aspects of knowledge acquisition processes were not really thematized by Piaget, the essentials of his theory enable us to deal with them. Moreover, it will be argued that Piaget's theory has a great heuristic value in the case of research concerning the psychogenesis of socio-cultural objects and in understanding its transformation into objects of knowledge.

Within the general framework of the theme 'Piaget and education', it is useful to retain a distinction between two closely related aspects:

(a) the relevance of social relationships to curricular learning (social relationships that are present in any process of learning in school settings);

(b) the learning processes specifically related to symbolic objects that are cultural products with high 'added social value'.

This chapter deals specifically with (b), taking as an example an object that has an enormous value in society and in school: the writing system.

Emilia Ferreiro (Argentina)
Graduate in psychology from the University of Buenos Aires. Ph.D. in psychology from the University of Geneva, under the direction of Jean Piaget (1970). Since 1979 she has been Full Research Professor at the Center of Research and Advanced Studies, National Polytechnic Institute, Mexico City. She is internationally known for her research on literacy development. Ph.D. Honoris Causa of the University of Buenos Aires. International Citation of Merit from the International Reading Association (1994). A UNESCO consultant on several occasions. Author of eight books and twenty-eight chapters of books in various languages: English, French, Italian, Portuguese and Spanish.

The characteristics of the problem

From a material point of view, any writing system (from its very beginnings until now) consists of a set of marks on a surface. Since their origins, human beings were producers of marks. It is even possible to assert that the intentional production of marks is a specific property of human beings.

However, not any collection of marks is writing. The specificity of the marks is irrelevant. It is of no importance if they were derived, as isolated marks, from drawings (progressively schematized) of actual objects. The important thing is that, at a given moment, those marks were organized in a system related to the language system. As the present evidence indicates, writing was invented only four times in human history, but in different chronological moments: in China; in Sumer; in Egypt; and in Mesoamerica (Michalowski, 1994). Two of those original systems were so successful and so apt to adapt themselves that they survive until the present day: the Chinese system, with relatively small variations throughout the centuries; and the Sumerian system, with profound modifications and various derivations, one of which is the alphabetic system.

Young children are also producers of marks. However, the crucial fact is that they need to understand the marks that other people, before them, have produced on various surfaces. These marks present themselves in a lineal order; they are organized in a space with well defined boundaries (the sheet of paper, for instance, that is the common surface of modern times). These marks look like a set of lines and strings between empty spaces, compact sets of marks, lines of various length.

These marks are impenetrable until an interpreter shows children that they have special powers: just looking at them produces language. No doubt, a language very different from that of face-to-face conversation, a language with a peculiar organization, with words that are not 'everyday words'. The one who reads speaks to another person, but what the reader says is not his/her own speech. The reader says the words of 'Another one' (or perhaps of many 'Ones'), that come from nowhere, hidden under the marks.

The transformation of the marks into symbolic objects takes place through the social practices of interpretation; these are the only ways to transform those marks into linguistic objects.

This transformation of impenetrable objects into transparent ones is not a one-day process. The reader reading aloud is making mysterious things; at the same time, the reader introduces the child into the labyrinths of mystery. To know that the marks allow the production of language is one thing. To understand how they do it is quite another. To arrive at an analytic understanding much more is needed, and much more time is required. The traditional teachers' discourses are far from being the key to the mystery. There are many—too many—children that pass an entire school year without understanding the teachers' discourse, based as it is upon the 'evidence' of an already literate adult. A

discourse that, in addition, is built on the simplistic idea that equates the minimal elements (the letters) as the easiest thing to learn, and the linguistically complex whole (the text) as having the greatest psychological difficulty.

To reach the analytical comprehension that links the written marks with oral language, something more than information is required: what is needed is the structuring activity of the knowing subject that Piaget has so well described in other domains. Specific information is essential, but, as always, the information will be assimilated by the subject in his/her conceptual schemes. A long and continuous interaction is also necessary with the object to be known, an object that presents itself through many other objects. In fact, written marks do not exist in isolation. The surfaces that the culture has invented to support (and transport) written marks have names and functions: they are called journals, books, calendars, identification papers, dictionaries, notebooks, announcements, commercial packages and containers, letters, etc. The list is endless.

Writing exists in contexts. The school blackboard is only one of the possible contexts: the most analytic, perhaps, but probably the least functional. Urban children growing up surrounded by print are not always surrounded by interpreters. All of them (including the children of illiterates) succeed in learning a great deal about those marks, thanks to their active efforts to compare them, to order them and to reproduce them. However, many children arrive at school without having had the experience of someone reading aloud to them, without having had the opportunity to be confronted with the challenge of disclosing the mystery, without having crossed the threshold of the opacity of the marks. Then, arriving to school, they face a teacher who, instead of acting as an interpreter, limits him-self/herself to the role of decoder, giving the name or the sound of each letter. Where here is the language? Where is the mystery, the challenge, the object to be domesticated?

Acquisition of knowledge does not follow from the presence of the object, as such; however, the absence of the object prevents the acquisition of knowledge about it. This is a basic principle that follows from Piaget's psychogenetic theory in relation to educational practices. If written language is absent from the school, if isolated marks replace written language as such, we will remain in the position that Vygotsky denounced already in 1935:

> The teaching of writing has been conceived in narrowly practical terms. Children are taught to trace out letters and make words out of them, but they are not taught written language. The mechanics of reading what is written are so emphasized that they overshadow written language as such (1978, p. 105).

We may be asked what is the purpose of presenting a citation from Vygotsky in a chapter dedicated to Piaget? Why do we relate Piaget with writing as a conceptual object, even though it is well known that Piaget never paid attention to such a cultural object?

Let us present a counter-argument: is it feasible to speak about 'Piaget and

education' without speaking about the most important of school acquisitions, i.e. those related to the acquisition of writing as a cultural object? During recent decades great debates have been pursued concerning the functions of the school and the content of the school curriculum. The only function that no one dismissed was the function of the educational institutions in creating literacy. Is it true that Piaget's theory is intrinsically unable to deal with knowledge acquisition of objects that were not the main concern of its author?

Piaget, social influences in the development and the knowledge of socially constituted objects

Even if Piaget did not thematized the psychogenesis of social knowledge, he did not show lack of sensitivity about the above-mentioned themes.

The deepest characterization of the notion of object—concerning the dichotomy physical object vs. social object—is to be found in the book *Psychogenesis and history of science* (Piaget & García, 1989):

> in the experience of the child, the situations she encounters are generated by her social environment, and the objects appear within contexts which give them their specific significance. The child does not assimilate 'pure' objects defined by their physical parameters only. She assimilates the situations in which objects play a specific role. When the system of communication between the child and her social world becomes more complex and enriched, and particularly when language becomes the dominating means of communication, then what we might call direct experience of objects comes to be subordinated, in certain situations, to the system of interpretations attributed to it by the social environment. The problem for genetic epistemology here is how to explain in what way assimilation remains, in such cases, conditioned by a particular social system of meanings, and to what extent the interpretation of each particular experience depends on such meanings (p. 247 of the English edition).

It is precisely in this text where the mixed nature of the object—a material object embedded in social meanings—is more clearly stated.

The importance of social factors in psychological development has been mentioned several times by Piaget throughout his extensive production. However, the mention of those factors does not replace their analysis. Piaget explicitly rejects a mere invocation to society that does not achieve at least the level of an analytic description as, for instance, in *Play, dreams and imitation in childhood* (1951), when Piaget explains his differences with Wallon. It is impossible to quote *in extenso* all these passages. Perhaps the following example will suffice :

> We are, of course, entirely in agreement that thought cannot be explained without recourse to social factors, but the general concept of 'social life' seems to us inadmissible in psychology. 'Society' is neither a thing nor a cause, but a system of relationships, and it is for the psychologist to classify these relationships and analyse separately their respective effects (p. 68).

We have chosen two quotations that belong to the two principal directions of Piaget's inquiries: he speaks of a problem that should be faced by genetic epistemology when dealing with the nature of the object at the very beginning of evolution; when he makes reference to the social influences he speaks as a psychologist. (It is relatively easy to find similar paragraphs in various of Piaget's books and articles.)

It is beyond our purpose to convince the reader that Piaget was concerned, one way or another, with social notions and with social aspects of scientific evolution.[1]

Rather, we will take the point of view of some of the most thoughtful of Piaget's critics. Their remarks could be schematically presented as follows:

1. Piaget acknowledged—perhaps 'malgré lui'—social influences, but he analyzed them as 'interferences'. Early in his work he anticipated a logical analysis of social relationships (in terms of 'constraint, imitation, discussion, etc.', 1951, p. 9), but he was never able to do this. He said that the operations are, in fact, co-operations, but he did not reach the level of demonstrating them.
2. Concerning social aspects, Piaget adopts a similar attitude to the one he took concerning neurophysiology: he expects coincidences instead of discrepancies between neurosciences and psychological analysis, but always insisting in the specificity of each domain. He recommended psychologists not to postulate, without sufficient evidence, imaginary neurobiological changes (as well as possible social influences) in order to explain the great psychological changes during development. No doubt, the latest will not be in opposition with the forthcoming discoveries of neurophysiology (or those of the social psychology, cultural psychology or whichever denomination one may choose).

From the first group of critics would follow the incapacity (or even the intrinsic impossibility) of Piaget's theory to deal with social objects.

From the second group of critics a methodological delimitation would follow of the specific research object (a delimitation entirely justified, even if subject to criticism). The door remains open to conduct empirical and theoretical research to prove the capacity of the theory to meet the challenges offered by other type of objects (neither physical nor logico-mathematical ones).[2]

We may admit that the characterization of the 'others' that belong to the immediate social environment of a growing child as 'particularly interesting objects' is entirely insufficient (or even inadmissible). However, the following two options remain open:

1. Either Piaget's theory applies exclusively to physical and logico-mathematical objects; or
2. Piaget's theory is a general theory of processes of knowledge construction developed around the problems of physical and logico-mathematical objects, but is potentially able to deal with the construction of other type of objects.

During more than two decades we have elaborated on the second position, a position that nowadays seems more difficult to promote because the new trends

favour the development of loosely co-ordinated (or even totally unco-ordinated) micro-theories. General theoretical frameworks are not 'à la mode'. Without leaving aside the necessity of theoretical debates, it is not irrelevant to ask: what are the discoveries, the new observables brought to light for those who maintain the possibility of general mechanisms? What are the new facts discovered by those who maintain the possibility of independent modular devices? (In both cases, in relation to the acquisition of social objects of high scholastic value.)

Let us give a brief description of our present knowledge of the development of writing, as a conceptual object during psychological development.

Literacy development: general and specific aspects

In many other publications we produced evidence showing that children make consistent efforts to understand the written marks of their culture. These efforts give way to a series of conceptual constructions that progress through a non-haphazard order. (This is true at least for writing systems built upon phonographic-alphabetic principles).[3]

Let us give a brief description of this conceptual evolution (impossible to present here in all its detail). After a period characterized by the search for distinctive parameters to distinguish between figurative marks and non-figurative ones, children's attention focuses on the distinction between elements and totalities. Now, their aim could be described as a search for the rules of composition that lead to potentially interpretable totalities (strings of letters where 'it could say something'). At that moment two fundamental principles are constituted: the minimum quantity principle and the internal variation principle. Following the quantitative principle, there must be a certain amount of letters (two at least; preferably three) for a meaning to be attributed, through a linguistic name, to a given string of letters. Following the qualitative principle, there should not be repetition of the same forms inside a given string. (This principle may present itself in a very strict way—no repetition at all—or in a less rigorous way—no repetition in a contiguous position.)

In the conceptual history of children's writing, the aspects related to the laws of composition and the way of organizing letter-strings that allow an interpretation (independently of contextual considerations) are prominent. Figurative aspects (the form of the letters) are less relevant.

Quantitative and qualitative variations will give rise, little by little, to the possibility of going from the consideration of each letter-string in isolation to a comparison among strings. Such comparisons will permit the establishment of whether two given strings may receive either the same or a different interpretation, on the basis of objective differences in their composition. It is very surprising to observe that, in order to build up a set of written strings with the above-mentioned restrictions, many children discover that a good way of going about it is to keep the forms constant but to change the position in the series (i.e. they discover

that an initial string OMA, for instance, could generate AMO, MOA, OAM, etc.) Of course, we are speaking of children that do not know how to read and write in the conventional sense of these terms.

Through genetic psychology we learned long ago that, in order to understand a given psychological aspect at the adult level, it is necessary to reconstruct its genesis. This is particularly true in the case of written language. Trying to understand this cultural object, children face combinatory problems (probably much earlier than in any other cognitive domain). As a result, written language becomes a cognitive space of problems. Learning about these kinds of combinations of a set of forms in a linear order, children discover a property specific to any writing system (including the alphabetical one): with a limited set of forms, it is possible to obtain, by combination, a great number of strings (constrained quantitatively and qualitatively by the two main principles already mentioned).

The phonetization period (i.e. the analytic link with oral language) is preceded by extensive cognitive work. The phonetization period begins, in all the languages already studied,[4] by a search of correspondence between letters and oral segments bigger than phonemes, syllables being the most suitable candidates. In spite of the fact that children are being exposed to alphabetic writing in the environment, and receiving alphabetic information at school, children construct a syllabic hypothesis (before or even at the beginning of primary school) where each letter (right or wrong) stands for a syllable.[5] (Of course, this kind of construction only become evident when children are allowed 'to write the best that they can, as they thought it may be', disregarding copying as the only way to have access to writing.)

It is not surprising that the main and more general problems of logical construction appear during the psychogenesis of writing as a conceptual object: co-ordination between similarities and differences; relationships between totality and its parts; one-to-one correspondence; problems of order and identity. It is not surprising—in Piagetian terms—because the logical schemes constitute at the same time the conditions of the 'reading of experience', as well as the result of the children's efforts for structuring the object of knowledge.

However, an important distinction needs to be established. Many psychologists and educators, inspired by Piaget's theory, have maintained a view according to which the only important objective in education is the development of operations, disregarding the content. In the case of 'unavoidable' school content (and written language is the most obvious of them!), they have tried to deduce what could be the operatory pre-requisites to start teaching it. (This is the case of Elkind (1976), for instance, who postulated that it is necessary to arrive at the period of concrete operations before starting to teach reading.)

The dichotomy is this: either the processes of structuring reality are constitutive of operational development; or the logical structures develop through objects, but 'any object', without apprehension of the content.

Careful psychological research leads us to assert that children start to deal with written material well before the school authorizes them to do so; that the

cognitive efforts deployed to understand written material also contribute to operational development; that, in addition to the already mentioned logical problems of general scope, there are specific problems to be solved with this particular object, mainly in its relations to oral language; that precisely because writing is a symbolic object, the interaction with the interpreters is not a mere addition, but that it is constitutive of the possibilities of understanding the nature of the object.

We have presented a very simplified view of the many problems concerning the acquisition of a writing system. The most traditional view considers the learning of reading and writing as the acquisition of a mechanical transcription technique. In sharp opposition to this view, we have been able to demonstrate that the writing system becomes an object of knowledge during development, an object that probably changes dramatically the relationship of the knower with his/her oral language. This new conception is heavy in educational consequences (impossible to develop here). To reach convenient standards of literacy it is necessary, in addition to a comprehension of the writing system as such, to have access to the enunciation conditions specific to written communication. It is necessary to learn how to tell, to argue, to ask, to answer, to solicit, to give instructions, to inform, to comment and to construct a dialogic situation through writing.

Piaget and the knowledge of cultural objects and social notions: a programme for the coming decades

The 'power' of a theory and its potential as an explanation are made evident, and their validity reinforced, when it is possible to apply the theory to domains that were not considered by the original author. This is linked to an important problem of construction of scientific theories: a theory does not explain isolated situations or events; it explains processes. As with physics, chemistry or biology, it is of no scientific interest to formulate *ad hoc* explanations for each particular kind of process. The acceptability of a theory depends on the range of phenomena it is capable of accounting for.

The research work in progress about the development of economic, political and social notions, taking into account a Piagetian framework, does not deny the specificity of these notions (see, for instance, Berti & Bombi, 1981; Delval, 1989; Furth, 1980). The most promising investigations are those that leave out the idea of a direct application of operations to new contents, and put emphasis instead on the structuration of reality in the process of operational development.

There are still many fields to be explored. For instance, in the domain of notations of social use, our knowledge of the psychogenesis of mathematical notation is still rather poor, as well as our knowledge of the acquisition of maps, diagrams and other notations of school use.

In all these fields, the contribution of Piaget's theory is of great value, not giving us ready-made answers, deductively obtained, but helping us to shape new research questions.

Nevertheless, to approach this goal, it is necessary to leave out the naïve view of graphic representational systems as mere 'figurative resources', a distorted reflex of operatory thought. Representational systems historically developed are not neutral: they help us to think in very specific terms about the represented object. In the case of writing systems, it is now evident that the resolution of problems specific to the acquisition of an alphabetic writing system is full of consequences about the way the speaker will conceive his/her own oral language.

From an epistemological point of view, it seems more and more urgent to consider a sociogenesis and a psychogenesis of socially developed systems of representation, with their own specificity, besides the history of the notions and the scientific explanation systems, as has been considered until now by genetic epistemology.

Notes

1. It is sufficient to remember that Piaget wrote a very influential book—particularly in English-speaking countries—devoted to the psychogenesis of social rules in collective play (*The moral judgment of the child,* 1968). He is also the author of *Epistémologie des sciences de l'homme* (1972), a text written for a UNESCO volume and never translated into English.
2. We are not taking into consideration here the arguments offered by some 'orthodox' Piagetians in favour of leaving aside social objects, on the grounds of an assimilation between social objects and arbitrariness. This kind of interpretation, presenting itself not as a criticism but as a justification, places social notions and social objects as entirely foreign to Piaget's theory, for intrinsic reasons.
3. The most accessible publications in English are: Ferreiro & Teberosky, 1982; Ferreiro, 1984, 1985, 1986, 1991, 1994*a*, 1994*b*.
4. Catalan, English, French, Hebrew, Italian, Portuguese and Spanish (in many dialectal varieties).
5. For instance, VKT for 'va-ca-tion' or SPM for 'su-per-man'.

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PIAGET AND SCHOOL EDUCATION: A SOCIO-CULTURAL CHALLENGE

Eduardo Martí[1]

From the outset, Piaget's work was a standard reference on educational research and practice. In the 1920s, his early studies on the mind of the child were hailed with interest in teaching circles (Parrat-Dayan, 1993). Later, in the 1950s and 1960s, in a favourable economic context and at a time when educational reforms were necessary to face the challenge of new technologies, the spread of his theory provided a backing for many of the attempts made to take a new approach to education. The undisputed importance of his work for education emerged from a theoretical study in which, paradoxically, his interest in teaching was only secondary. It is true that between the late 1920s and the 1970s he published a score of articles dealing with education. This interest arose in part from his responsibilities as Director of the International Bureau of Education from 1929 to the 1960s. Articles in scientific journals and a series of speeches to international conferences on State education dealt with a wide range of educational issues such as the psychological bases of a new kind of education, the scientific bases of education in the future, education for peace, freedom, art, modern mathematics teaching, history teaching, teamwork at school, etc. (For further information the reader should consult the complete bibliography published in 1989 by the Fondation Archives Jean Piaget.) But this activity, which brought Piaget into contact with the problems of education, is somewhat secondary in relation to his real interests. Despite the possibility of 'helping to improve teaching methods and gain official recognition of techniques more suited to the mind of the child' (Piaget, 1966, p. 10), he viewed his involvement in the routine administration of education ironically and from a distance: 'This certainly took up much of my time, which could

Eduardo Martí (Spain)
Professor of developmental psychology at the University of Barcelona. Doctor's degree in psychology at the University of Geneva, where he worked with Jean Piaget at the International Centre of Genetic Epistemology. Research on mathematical problem-solving by children and the use of computers as a learning tool.

probably have been spent more usefully researching into child psychology; however, it did teach me a great deal about adult psychology' (ibid.).

Whereas he wrote extensively on epistemology and psychology, Piaget published only two studies directly concerned with education: *Psychologie et pédagogie* (1969) and *Où va l'éducation?* (1972). And though he published some works on the problem of learning as such, his approach was highly individual and at all events far removed from most traditional studies. In refuting empiricist positions, he argued that it was impossible to explain new acquisitions without having recourse to inner regulatory mechanisms (Goustard et al., 1959; Morf et al., 1959). For Piaget, it was important to highlight the dynamic role played by the equilibration of all acquisitions, even those induced by experience. Here his studies are far from concluding that learning mechanisms in general, and school learning mechanisms in particular, have any specificity in relation to general developmental processes. It is the latter, and the attendant constraints depending on the child's level of development, that determine the next stage of learning abilities. One can therefore understand his mistrust of any deliberate attempt to speed up the acquisition of a concept or an operational structure:

> For each subject, the speed of passage from one stage to the next probably corresponds to an optimum, neither too slow nor too fast, with the robustness and even fruitfulness of the new organization (or structure) depending on connections which can be neither instantaneous nor indefinitely retarded without losing their power of internal combination' (Piaget, 1968, p. 290).

Despite the secondary place given by Piaget to educational considerations in his work, his main aim being to elucidate epistemological problems, and despite his subordination of learning processes to specifically developmental processes, Piaget's constructivism, as noted above, was for many years decisive for all those who sought to rethink education. However, in recent years serious criticisms based on cognitive psychology, and above all socio-cultural theories, have identified the limits of his constructivism in explaining the complexity of educational practice.

Strengths and weaknesses of Piaget's constructivism

THE CHILD, THE SCIENTIST AND THE PUPIL

There is no doubt that Piaget's findings on the child mind helped to pinpoint children's abilities and limitations at different stages of development as regards learning at school. His findings on the sequence of acquisition of physical concepts (weight, volume, density), space and geometry (order, co-ordinates, the straight line, surface, volume), mathematical and logic (seriation, classification, number, proportion) provide an invaluable reference point for educational practi-

ce. They give a justified basis for the choice of programme content, and offer various models of teaching sequences.

But this borrowing of information from genetic psychology has inherent difficulties when applied to educational practice. It is based on a general model for the construction of knowledge which is valid not only for the development of the child but also for the construction of scientific knowledge and the acquisition of knowledge at school. In other words, in seeking primarily to discover what is common in the way children construct knowledge, above all scientific knowledge—either spontaneously in everyday life, at school or within each scientific discipline—Piaget ended up with an explanatory model common to all three types of knowledge (spontaneous, scientific and that acquired at school).[2] However, everything suggests that a distinction should be drawn between these three types of knowledge.

Many studies, for example on cognitive and educational psychology, show that at all ages children build up a series of domain-specific types of knowledge: number, biology, classification, the structure of matter, movement, causality, social relations and categories, etc. (Driver, Guesne & Tiberghien, 1985; Hirschfeld & Gelman, 1994; Rodrigo, Rodríguez & Marrero, 1993). This knowledge comes from the child's interaction with everyday situations, and is thus not constructed in a formal teaching context such as the school. It consists not of isolated representations but genuine networks of concepts and principles which guide the child in the way he foresees and reacts to the various problems facing him. Everyday knowledge differs profoundly from that taught at school. While the former is usually implicit, the latter is explicit in nature; the former is often incoherent and mindless, the latter coherent and consistent; the former is often the result of an inductive process which includes verification strategies, the latter is deductive, reached by strategies of refutation; everyday knowledge lumps together covariation with the cause and effect relation, while knowledge learned at school is able to distinguish between them (Rodrigo, Rodríguez & Marrero, 1993, p. 85).

Again, everyday knowledge is of the type which is far from being easily modified by instruction. It is usually highly resistant to change and modification, and where its integration to school-acquired knowledge is possible, this is one of the major challenges of education today. There is no doubt—as Piaget shows—that the child, coming to school for the first time, is not an empty receptacle or 'tabula rasa', but will assimilate new knowledge into his own cognitive patterns and structures. However, as shown by the studies quoted above, the assimilating context does not consist only of patterns and structures which change spontaneously with the child's development; it also comprises implicit types of knowledge which are obviously used to solve everyday problems, and which have great difficulty in changing except as part of a conscious explanatory process guided by an adult. When becoming a pupil, a child must replace or restructure his implicit knowledge, which is very different from knowledge learned at school and is built up in equally different contexts. Examples are mathematics, physics, bio-

logy and history. Some of these types of knowledge are constructed very early on, during a child's first years, which means that a four or six-year-old begins school with a rich store of explanatory ideas and principles on number, space, objects, time, etc. (Gardner, 1991). Throughout their school life, children continue to construct implicit types of knowledge in interaction with their physical and social environment. This is decisive for what they learn at school. If we do not distinguish between school knowledge and other types of knowledge built up in less formal contexts, we run the risk of overlooking school-related specificities by comparison with other everyday contexts. At school, where the model is scientific knowledge, learning contents and objects differ greatly from those of everyday life. Activities are organized on diametrically opposed lines. Where the school looks for problems to solve, a child struggles with problems not of his seeking. School knowledge aims at precision and rigour, and is slow and cerebral, whereas everyday problems are often ill-defined, ambiguous, need to be solved urgently and arise in situations strongly charged with emotion (Claxton, 1991). These differences must be borne in mind, not so as to oppose the child to the pupil, or everyday knowledge to school knowledge, but so as to recognize their specificities and thus ensure their better integration, or the peaceful existence side by side of both types of knowledge.

By comparing school knowledge with everyday knowledge, we have implicitly equated school knowledge with scientific knowledge. Science subjects are certainly those that basically determine the choice of contents and objectives of school learning. But we should also note that between science subjects and teaching content exists a complex process of selection and filtering called by Chevallard 'la transposition didactique' (Chevallard, 1985). In this process, the carving up required to prepare school programmes detaches fragments of knowledge from their scientific context. This may lead to teaching contents which have lost the significance they had in the scientific conceptual context and which acquire new significances which are inseparable from educational practice. If therefore, as suggested by Piaget, we identify the scientific approach with that of the child (or pupil) by noting that they have shared acquisition mechanisms, the result is an image of school learning which is disembodied and deprived of its context. Whereas over the centuries scientists have always constructed their knowledge in a determined socio-cultural context, the pupil is confronted with a series of scientific contents prepared in advance and cut off from the context in which they were created; all this is information which he must reconstruct (rather than 'construct') in a special educational context quite different from the context specific to scientific creation. Awareness of these differences can help us to identify more precisely the obstacles encountered by pupils in the acquisition process.

CONTENTS AND ACQUISITION MECHANISMS

There is no doubt that one reason for the attraction felt for Piaget's psychology in educational circles lies in the dominating place assigned to the subject in the lear-

ning process. Piaget's work is marked throughout by the aim of specifying the psychological mechanisms characteristic of this activity of the child. Unlike other theories of knowledge which are more empiricist or emphasize innate qualities, Piaget's constructivism assigns to the child's action a preponderant role in creating new forms of knowledge which from the outset are found neither within the child (the theory of innatism) nor in his environment (the empiricist theory) (Piaget, 1970). From his early studies onwards, Piaget argued that the subject actively explores his environment and uses his actions to create internal structures which enable him to learn about the world in an increasingly objective and differentiated way. But it would be banal to conclude from Piaget's work that account should be taken of the pupil's activity in order to give an account of his school learning processes. This idea may have been important at the beginning of the century in educational circles still too close to an educational model based on the idea of the transmission of knowledge, but today it is part and parcel of any educational proposition. What is important from Piaget's point of view is the indissociable relationship between the learning subject and the object about which he is learning: the subject constructs internal structures and in this way creates the essential conditions for learning about objects. This relativist and non-dualistic position is different from a number of propositions which, while postulating the need for the subject to be active in the learning process, suggests a clear separation between the subject and the reality to be learned, or a static relationship between the two (Karmiloff-Smith, 1985; Sinclair et al., 1985). What is characteristic of Piaget's constructivism is precisely the argument that the subject constructs new knowledge while continually transforming his relations with the world: he changes both his way of thinking and the world he is getting to know (Piaget, 1980*a*). This view is entirely different, for example, from that which argues that the child does no more than appropriate or internalize types of knowledge which have already been constituted and can be found in his surroundings, or the view that the construction of knowledge is no more than the unfolding of abilities which already exist within the subject.

Faithful to his epistemological interests and the search for explanatory principles to account for what is common in the construction of rational thought, Piaget adduced a series of general mechanisms to explain the driving force behind this activity of the subject. Regulating processes primarily specific to equilibration, but also reflective abstraction, growing awareness and generalization were the psychological mechanisms used by Piaget to explain the construction of knowledge as it emerges from psychogenesis, and even from the history of science. The same mechanisms apply to all types of knowledge regardless of context. Thus general mechanisms gain the upper hand over more specific mechanisms.

Many studies conducted in recent years on cognitive psychology largely refute this general view. They show the importance of domain-specific restrictions (space, time, language, number, persons, etc.) in respect of an understanding of the process in which knowledge is constructed in each of these domains (Brown, 1990; Carey & Spelke, 1994; Hirschfeld & Gelman, 1994; Karmiloff-Smith,

1992; Keil, 1981). From this point of view, the subject's construction process depends on the principles specific to each of these domains and not only, as argued by Piaget, on the general principles related to the momentum of his actions, and how they are co-ordinated.

This view has consequences for educational research. If, for example, one seeks to study the construction of numerical knowledge by singling out moments in time and guiding principles, it is not enough, as suggested by Piaget, to refer to operational activity and the co-ordination of classification and seriation abilities. One has to discover how certain number-specific principles (terminological correspondence, cardinality, the non-relevance of the order of counting, composition by addition, etc.) guide the child's construction of a knowledge of number and his abilities to understand (Gelman & Gallistel, 1978; Resnick, 1986). This has led to a renewal of psycho-pedagogical and instructional research oriented towards a study of the acquisition of specific contents (which are closer to school programme contents).

But the importance of the specificities of school learning processes is not incompatible with the existence of general mechanisms for the construction of knowledge. As noted by Karmiloff-Smith (1992), the restrictions inherent in the various fields of knowledge provide an initial groundwork for adaptation which is based on the automatic nature of the information processing linked to specific inputs. Cognitive development is also characterized in its later stages by greater flexibility within each domain and the establishment of new relationships between the knowledge of different domains. It is therefore just as dangerous to postulate an acquisition of knowledge based solely on general principles as it is to refer only to specific principles. One should constantly bear in mind the interaction between the two types of mechanism and also take into account both the acquisition specificities of each domain and the general mechanisms (abstraction, awareness, equilibration) which are necessary to explain the trend towards the construction of increasingly flexible, explicit and general types of knowledge.

THE SOCIO-CULTURAL NATURE OF LEARNING AT SCHOOL

We noted above that in turning from the child to the pupil, it is necessary to go beyond Piaget's spontaneous constructivism (which gives an account of the acquisition of the basic universal structures of thought) and adopt a viewpoint which sees the pupil as a re-constructor of specific types of scientific knowledge selected as school programme contents for their cultural value. From this point of view the construction of knowledge in the school context requires that account be taken of a cultural and social dimension not to be found in Piaget's postulates. This socio-cultural dimension comprises two essential closely linked aspects, the mediating nature and the social nature of knowledge.

Semiotic mediation

For Piaget, the appearance of the semiotic function is a crucial time in cognitive

development. Thanks to this function, intelligence becomes representative: actions and their co-ordination can be carried out on a new internal plane and can thus be detached from external data. This means a gain in the subject's autonomy and control (Piaget, 1936). This new representative ability is possible because intelligence is now based on symbols and signs which in turn are explained by their being related to imitative activity (Piaget, 1946). However, notwithstanding the importance attached by Piaget to semiotic mediation, signs and symbols are no more than a medium and instrument of thought. They promote the acquisition and recording of information but have little in-depth effect on the nature of cognition. What is really new in the appearance and functioning of the semiotic function is in Piaget's view the properties emerging from the dynamic of actions and their co-ordination (greater flexibility and greater powers of combination thanks to the equilibration process).

For some years now, Piaget's view has been questioned on two lines. On the one hand, a number of investigations on the development of semiotic and notational systems in the child (language, writing, mathematical notation, drawing) indicate the specific nature of the constructive mechanisms of each of these systems (Karmiloff-Smith, 1992; Teberosky, 1993; Tolchinsky & Karmiloff-Smith, 1993). In this way, the child learns very early on to differentiate between written language and mathematical notation, and applies entirely different principles when it comes to producing one or the other (Tolchinsky & Karmiloff-Smith, 1993). Contrary to Piaget's argument, it seems that the development of semiotic systems should not be viewed as an application of the characteristics of general operational competence to different semiotic domains. On the contrary, each of these domains has its own acquisition mechanisms and principles.

Furthermore, some studies show that semiotic systems not only constitute a support for thought but that they deeply modify its functioning. Whether it be writing, mathematical notation, drawing, images, or informatics, these studies indicate the influence of semiotics on the cognitive function (Goody, 1987; Greenfield, 1984; Olson, 1986; Salomon & Leigh, 1984). The argument is that a cognitive activity which does not involve semiotic mediation would be profoundly different from an activity which does involve it; and cognitive activities would differ depending on the type of mediation. Thus an understanding of the construction of knowledge should envisage not only the general characteristics specific to operational activity but also those due to the type of mediation involved.

This view, which ascribes a fundamental place to semiotics in the construction of knowledge, is close to some of the postulates in Vygotsky's theory which accord to the use of semiotic systems an important role in cognitive development. For Vygotsky, the fact of using different semiotic systems entails a difference in the content and form of thought (Van de Veer & Valsiner, 1991). In addition, it is the semiotic character of activities (carried out to begin with in the context of dynamic interpersonal relations) which makes it possible at a later date to internalize these activities in the form of individual thought. Vygotsky considers that

semiotic systems should be understood as cultural creations passed down from generation to generation, used by adults and gradually integrated by children as they develop. This is a view which gives semiotics an educational role not to be found in Piaget, and also a view which accords primary importance for a child's cognitive development to the cultural instruments in his surroundings. Thus we have a child (pupil) who not only uses his internal abilities when tackling learning at school, but who also uses culturally constructed semiotic systems which are necessary for this learning and which radically alter his way of thinking and learning. From this point of view, school learning mechanisms must be seen as activities for the construction of knowledge which are closely bound up with the cultural instruments used by adults and made available to children.

Social interaction

Piaget's view of the construction of knowledge gives a preponderant role to individual internal mechanisms. Piaget never denied the importance of social factors in explaining cognitive development. In his early studies he demonstrated the key role played by social interaction in the change-over from egocentric thought to more logical and social thought, and the importance of the comparison and co-ordination of viewpoints in the child's cognitive progress (Piaget, 1923, 1924, 1932). Later on, his studies barely touched on the dynamic role played by external forms of social regulation. These are self-regulating mechanisms which are decisive for explaining the form and direction of cognitive development. It is true that inter-psychological activities (those which emerge from interpersonal interaction) are seen by Piaget as constituting a whole articulated with the subject's internal actions. However, in the same way as he makes semiotic mediation subordinate to operational activity, he subordinates inter-psychological activity to the operational progress, and the general mechanisms, of the co-ordination of actions:

> All logical thought is socialized because it implies the possibility of communication between individuals. But these interpersonal exchanges proceed from correspondences, additions, intersections and reciprocities, that is to say, through operations. There is thus identity between the intra-individual and inter-individual operations which constitute *co-operation* in the strict and almost etymological sense of the word. Whether actions are individual or interpersonal, they are by their essence co-ordinated and organized by the operational structures constructed spontaneously in the course of mental development (Piaget, 1985, p. 137).

The subordination of inter-psychological actions to the subject's internal constructive dynamic does not help to explain the workings of the construction of knowledge in the school context, where the pupil's social partners (the teacher and other pupils) are basic partners in the acquisition of knowledge. Vygotsky, with others belonging to the same theoretical tradition, proposes a view which

brings us closer to the social dynamic of the school context. For Vygotsky, it is precisely the characteristics of interpersonal action that determine the main aspects of internal functioning. These characteristics include the semiotic nature of communicative interaction, which Vygotsky considers vital for the understanding of the relation between inter-psychological and intra-psychological functioning (Vygotsky, 1981*a,* 1981*b*).

The interest of Vygotsky's position for a constructive theory of learning at school is twofold. First, it enhances the role played by interpersonal interaction by postulating a profound articulation between the inter-psychological and intra-psychological planes. Secondly, what explains this isomorphism between inter-psychological and intra-psychological organization, and also explains the possibility of internalization, is the semiotic nature of inter-psychological activity. Thus the child's contact with the external forms of activities (which use semiotic mediation) explains how the child is able to create new forms of internal activities.

The importance accorded by Vygotsky to sign systems (the products of cultural development) in the construction of knowledge, is in agreement with the research findings quoted above, showing that the use of semiotic systems entails fundamental differences in cognitive functioning. Similarly, the importance accorded by Vygotsky to inter-psychological activity in the construction of intra-psychological activity is illustrated today by many studies which indicate the importance of different inter-psychological mechanisms in cognitive functioning and the acquisition of new knowledge, in both formal and informal learning contexts (Resnick, Levine & Teasley, 1991; Rogoff, 1990). There is no doubt that we draw nearer to the reality of school learning processes when these are viewed as processes involving not only the child's structuring activity but also the interactive mechanisms which enable the child to reconstruct a teaching content selected in advance. On the one hand, there are the teaching mechanisms whereby the teacher creates the conditions in which it will be possible for him continually to adjust his educational assistance and in which it will be possible for learning to take place: for example, the gradual transfer of control from the teacher to the pupil and the gradual construction of shared meanings (Coll et al., 1992; Coll & Onrubia, 1994). On the other hand, there are the mechanisms of inter-pupil interaction which also play a fundamental role in problem-solving and in the process of acquisition of knowledge at school: the explanation of information, confrontation of viewpoints, sharing of the cognitive burden (Brown & Palincsar, 1989; Forman, 1989; Martí, 1994*b;* Schoenfeld, 1989).

Thus the idea of a pupil who spontaneously constructs the cognitive structures which enable him to accede to the major contents of school programmes should be reformulated on two lines. First, account should be taken of the specificity of curricula contents (themselves the result of a selection of scientific contents) which are learned by means of mechanisms which cannot be reduced to the general mechanisms noted by Piaget. Second, one should also analyse the acquisition process in close articulation with the interactive processes which take place in the school context.

Internalization and externalization at school

The limits to Piaget's theory noted above should not lead us to replace his postulates by those of other theories, for example those elaborated in the tradition of Vygotsky's socio-cultural theory. This might well lead us to propositions which would be more suitable than those of Piaget for giving an account of the social dynamic inherent in any constructive process (for instance at school), but which would certainly fail to account for the individual's internal dynamic and its structuring role in learning at school. One might also be tempted to juxtapose certain of Piaget's postulates and those emerging from other theoretical contexts (such as that of Vygotsky) in an attempt to overcome the limits of all these postulates. In our view, this might lead to an eclectic position in which unreconcilable arguments would be juxtaposed and would in the end lose any explanatory value.

We shall therefore start from the basis of the postulates of Piaget's constructivism, but reformulating them on some of the lines noted by the studies we have just quoted. This re-interpretation of Piaget's arguments with a view to obtaining an explanatory context more suitable for giving an account of learning at school, will be linked to two processes which are essential for the construction of knowledge: the internalization and the externalization processes (Martí, 1994*a*).

AN INTERNALIZATION PROCESS

Throughout his work, Piaget defended the importance of the internalization process in explaining the constructive character of knowledge. When discussing the change from sensorimotor to representative intelligence, he showed that the latter was not merely the product of a transposition of external knowledge to an internal plane but actually constituted a new level of functioning (Piaget, 1947). This new level results from the greater speed with which schema combine together. Thus the structuring activity no longer needs support from either the successive ongoing data of perception or continuing external control. In Piaget's view this is possible thanks to the new ability to represent (the ability to evoke absent objects) (Piaget, 1936).

Despite the importance accorded by Piaget to the change from sensorimotor to representative intelligence, the internalization process is not limited to this change. He considered that the trend to internalization was present throughout development. In fact, cognitive development is characterized precisely by a gradual conquest of internal autonomy (the result of continuing co-ordination of actions) at the expense of the dependence of the intelligence on external data closely related to perception. Thus the acquisition of knowledge consists in a recurring passage from knowledge of an external nature (linked to observable elements and to perception) to more endogenous types of knowledge. Piaget highlighted this trend in his final studies on the functional mechanisms of the construction of knowledge (Piaget, 1980*a*). Both the relations between empirical

and reflecting abstraction and the relations between inductive and constructive generalization, or the relations between correspondences and transformations, clearly indicate this trend from the exogenous to the endogenous. It is certainly the study of awareness which brings out most clearly this general trend towards internalization, since the awareness mechanism itself works in a direction from the more peripheric to the more internal aspects of action (Piaget, 1974).

This general trend towards internalization is linked to the actual equilibration process. We know that Piaget ascribed to desequilibria, and the resulting compensatory equilibration process, a primordial role in the construction of knowledge. Precisely, the two main classes of disequilibrium, external (difficulties in applying and attributing schema to objects) and internal (difficulties in combining these schema) give a reliable account of the interaction between the exogenous and the endogenous (Piaget, 1975).

The importance accorded to the internalization process is equally manifest in the work of Vygotsky and the authors who adhere to the postulates of socio-cultural theory. However, these authors attribute a more important formative role than Piaget does to external functioning (which is for them of a social nature). Vygotsky considers that all psychological processes are formed at two separate levels: firstly on the social plane (inter-psychological functioning) and secondly on the individual plane (intra-psychological functioning):

> it is necessary that everything internal in higher forms was external, i.e., for others it was what is now is for oneself. Any higher mental function necessarily goes through an external stage in its development because it is initially a social function. This is the center of the whole problem of internal and external behavior. [. . .] When we speak of a process, 'external' means 'social'. Any higher mental function was external because it was social at some point before becoming an internal, truly mental function (Vygotsky, 1981b, p. 162).

We thus see that, unlike Piaget, Vygotsky defines the external plane as that constituted by social interaction. Here he clearly defends the social origin of the construction of knowledge. However, as noted by Wertsch (1985), it is not a question of interpreting the social origin of knowledge as meaning that individuals learn through their participation in exchanges with other people. For Vygotsky, there is more than this. There is a profound connection between the two functioning planes, with the external plane determining the main aspects of internal functioning. It is precisely the internalization process which makes it possible to achieve this connection. Internalization transforms social phenomena (seen by Vygotsky mainly as interpersonal interaction) into psychological phenomena which guard the main characteristics of the former.

The internalization process as seen by Vygotsky cannot really be understood without the input of semiotic mediation. This mechanism helps us to grasp the very close connection linking the social and semiotic nature of higher psychological processes. For Vygotsky, these processes, unlike elementary processes, are those which use signs. The incorporation of signs in psychological activity (for

example the use of linguistic signs in memorization) does not only promote this activity, but modifies it profoundly when it is mediatized by signs. In Vygotsky's view, the key to an understanding of the forms of internal semiotic mediation is to be sought in the social and external origins of signs. And this has a twofold meaning. Sign systems (language, counting systems, algebraic symbol systems, mnemonics, etc.) are of a social nature since they are the product of socio-cultural evolution and are not invented by each individual in his relations with nature; they become individual, that is to say internal to the functioning of each individual, precisely through the internalization process. In addition, signs have a social nature in that the sign emerges in the communicative dynamic of social interaction. Vygotsky views the sign as a means originally used for social reasons which enables one to influence others, and it is only later on (thanks to internalization) that it becomes a means of influencing oneself (Vygotsky, 1981*a,* 1981*b*).

It seems undeniable that any cognitive acquisition implies an internalization process. Piaget and Vygotsky agree on this general postulate, but they each see the nature of the process differently. While for Piaget internalization is a process which renders knowledge increasingly autonomous and independent of external data, for Vygotsky it is the passage from external, social functioning to internal, individual functioning. This is possible thanks to semiotic mediation. In attempting to understand the acquisition mechanisms specific to educational situations, Piaget is interesting in that he explains the internalization process by adducing individual mechanisms (awareness, abstraction, equilibration), which are the key to an explanation of the new constructive nature of acquisitions. These mechanisms are not to be found in Vygotsky's work. But Piaget's position is limited by the fact that neither social interaction nor semiotic mediation plays a formative role in this acquisition process. The chief input of Vygotsky's postulates to Piaget's constructivism is the new emphasis on semiotic mediation and the social nature of all forms of acquisition of knowledge. These two dimensions are vital in the educational context.

THE CONSTRUCTION OF KNOWLEDGE ALSO IMPLIES A PARALLEL EXTERNALIZATION PROCESS

The internalization process, vital for an understanding of the construction of new knowledge in general and school-acquired knowledge in particular, is in turn inseparable from a parallel externalization process. Piaget sees the externalization process as an ever-deepening understanding of the properties of objects and their relationships. Thus, while through internalization the subject constructs increasingly stable internal types of knowledge which are also more mobile and removed from the immediate perception of data, with the externalization process the subject deepens the properties of objects and the relation between them. In fact, this dual internalization/externalization process is inherent in Piaget's interactionism, which is characterized by a twofold construction: that of the internal structures of thought and that of the ever-deepening knowledge of external reality (Piaget, 1980*b*).

However, as suggested by arguments on the lines of Vygotsky, externalization can also be understood as a process of fleshing out knowledge which has previously remained latent. Piaget himself, in his studies on awareness, suggests that there is such a process: a construction of increasingly explicit types of knowledge which the subject can externalize by means of gestures (when he simulates what he has just done) or by verbalizations (when he explains what he has just done) (Piaget, 1974). In fact, Piaget recognizes that awareness generates conceptualizations at different levels of explicitness ranging from a barely outlined awareness of success after self-regulation to manifest and clearly verbalized states of awareness.

Notwithstanding his divergences from Piaget's position, Karmiloff-Smith also argues for the existence of a general recurrent mechanism, 'representational redescription', which is responsible for the passage from subconscious, implicit and function-related knowledge to conscious, accessible knowledge expressed by verbal explanations (Karmiloff-Smith, 1992). This transformation of knowledge, linked to the stability and success of cognitive functioning rather than to failures or conflict, comprises at least three levels: a level of implicit knowledge represented procedurally, a second level defined explicitly but which cannot be verbalized, and a third level which can be verbalized.

On the same line of thought, Allal and Saada-Robert (1992), basing themselves partially on Piaget's typology of regulating mechanisms, distinguish between four more explicit degrees of cognitive regulation: (1) implicit regulation, integrated into the cognitive function, of which the subject is not aware; (2) regulation accessible to awareness and which can be rendered explicit in the case of an external demand; (3) explicit regulation, conscious and communicable to other people; and (4) regulation by instrumental means based on a support external to the subject's thought. In the last case, which is of special interest to us, instrumentation may be based on a support produced by the subject himself (the outline of a text, a diagramme, a mnemonic notation, etc.) or a support provided by another person (a list of criteria provided by a teacher, a diagramme from a computer programme, algebraic symbols, etc.).

The interest of all these propositions is that they make possible a more detailed approach to the externalization process and ascribe to it a dynamic role in the construction of knowledge. Thus externalization would not be merely (as argued by Piaget) a gradual construction of external reality, but also a re-organization of knowledge on the lines of a conscious gradual process of achieving greater explicitness. In this process, the social context would play a decisive part. In its turn the process entails a greater possibility of communication and sharing with others. As knowledge is explicitly constructed (and is based on signs or symbols frequently materialized in external representations or specific symbolic notations), it has a feedback effect on cognitive functioning; its communicative potential also increases. It is obvious that seen in this way, the externalization process plays an important part in the construction of knowledge and is closely linked to semiotic mediation and social interaction.

Conclusions

Piaget's theory proved decisive for an understanding of certain psychological mechanisms involved in learning at school and for suggesting teaching and learning sequences of scientific contents on the basis of an epistemological analysis of the main scientific notions. Notwithstanding this essential input into educational reflection, Piaget's theory has a number of limitations which are linked to its inability to give an account of the specificities of the school context and its specific acquisition processes. By way of conclusion, we recapitulate the main points developed in this article.

1. Psychological mechanisms such as awareness, abstraction or equilibration are essential for an understanding of the constructive nature of learning at school. They enable us to analyse a pupil's structuring activity and thus to distance ourselves from all educational options which do not define the constructive and creative nature of the subject. However, many studies in a variety of domains (mathematics, science, reading, writing, social skills) have shown the importance of the specific knowledge built up by subjects spontaneously during their interaction with their physical and social surroundings. In addition, the mechanisms involved in acquiring knowledge in these different fields are in part specific. They are not reducible to the progress of the operational activity. It is therefore necessary to go beyond Piaget's constructivism and accept the importance of domain-specific acquisition mechanisms.
2. When considering the possibilities that the human species might achieve rational thought, Piaget concentrated on the common and general points of cognitive activity and underestimated particularities as regards individuals or contexts. It is therefore not surprising that in viewing the child, the pupil and the scientific context, Piaget saw only an epistemic subject regulated by the same construction mechanisms. Yet from the viewpoint of educational practice, it is vital to analyse their differences. Many research findings which compare knowledge constructed by pupils in the school context and knowledge constructed by the same pupils in different socio-cultural contexts, bring out their differences in detail (degree of accessibility, functionality, degree of organization, disparity of strategies). Educational practice takes place in a special context which partly determines the characteristics of academic knowledge. Analysing the specificities of this learning context and at the same time taking into account the implicit types of knowledge acquired by pupils in other everyday contexts, appear to be two key objectives if we are to go beyond Piaget's epistemic subject and approach the 'educational subject'.
3. One of the main features of the school context is its social nature. While considering that social interaction is a factor necessary for the construction of knowledge, Piaget never specified its structuring role. He subordinated it

to general self-regulating mechanisms. Yet if we are to tackle school learning processes, we must ascribe a central role to other people's regulating mechanisms. On the one hand, the mechanisms of the supportive and guiding activities of teachers are essential for an understanding of the way in which pupils reconstruct the cultural knowledge selected by the school. On the other hand, the mechanisms which regulate inter-pupil interaction are essential for an understanding of how knowledge learned at school becomes increasingly more explicit and shared.

4. The other main feature of the school context is the mediating nature of teaching and learning activities. In Piaget's view, semiotic mediation, although essential to guarantee the representative basis of thought, is dependent on and subordinated to operational activity. Yet many studies show on the contrary the specific importance of mediation and its impact on cognitive functioning: language, writing, mathematical notation and the use of a variety of cultural instruments play a decisive part in the construction of what is learned at school.
5. While it is essential to see the construction of school knowledge as a recurrent internalization process whereby the pupil builds up knowledge which becomes increasingly autonomous and less dependent on external and perceptive aspects, it is also necessary to consider the reciprocal process of externalization. It is through this process that knowledge becomes gradually more explicit and can be made verbal, hence more suitable for being communicated and shared by other social partners.

Piaget, somewhat in spite of himself, was decisive for educational research and practice. He will continue to be so if constructivist postulates integrate the main challenges from contemporary research, which is more open to domain-specific knowledge and the specificities of the semiotic and social mechanisms involved in the school context.

Note

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2. At the Geneva School, a number of recent theoretical developments show the need to proceed from a study of the 'epistemic subject' envisaged by Piaget (what is common in the cognitive structures of subjects at the same level) to a study of the 'psychological subject', and thus to give greater emphasis to aspects linked to the dynamism of individual psychological functioning (Inhelder, 1978; Inhelder et al., 1992).

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MATHEMATICS TEACHING FROM THE STANDPOINT OF GENETIC EPISTEMOLOGY

Gisèle Lemoyne[1]

> *To know an object is to act upon it and transform it, in order to grasp the mechanisms of the transformation together with the transforming actions.*[2]
>
> JEAN PIAGET

In his book *Psychologie et pédagogie*, Piaget (1969) reviewed the evolution of the relations between psychology and education from 1935 and wondered why the 'profound renewals in child psychology' (p. 12) had had such little impact on education sciences. He observed that, for several education scientists, the main advantage of active methods 'is to replace abstract ideas with concrete connections. [They] even aim to reach the forefront of education by increasing the number of intuitive representations in forms which are no longer active in any way' (p. 56-57).

This observation still seems relevant to us now, since several methods claiming to be inspired by genetic epistemology entail this kind of simplification. It should be further noted that in recent decades we have witnessed an increase in

Gisèle Lemoyne (Canada)
Professor in the Department of Didactics of the University of Montreal, where, after undergraduate studies in science and psychology, she obtained a Ph.D. in psychology in 1976. She wrote her doctoral thesis, on computer modelling of the psychological processes involved in the development of seriation, under the supervision of Professor Georges Baylor. After this initial research in psychology, she became interested in the didactics of mathematics. Several of her studies on the processes of learning and teaching arithmetic and elementary algebra have been carried out in computer micro-worlds. Her research work has been published in American and European journals.

physical and figurative objects 'connected' with the concepts being taught, and the introduction of new curricula. These objects and curricula are very often based on research in educational psychology and didactics, that is when the researchers themselves have not produced them.

Several interpretations of this simplistic relationship between psychology and education have been suggested. Vanderdorpe (1992) has emphasized that one begins to interpret a text or work by situating it in relation to the author's works as a whole and to reference books. The interpretation also depends on the relations the reader establishes with the work according to his previous knowledge or objectives. These contributions by the reader can distort or betray the text—we speak then of an ideological effect. In the case of works by Piaget and his co-workers at the International Centre for Genetic Epistemology, it is easy to understand that their range and complexity have hardly facilitated their interpretation.

The following interpretation, suggested by Brun (1993), is a good description of the work involved in reading the Piagetian corpus:

> The proximity of concerns about the transformation of knowledge by a subject or a pupil probably explains why work on cognitive development psychology is used directly, without questioning the nature of the purposes and subject-matter of psychology and didactics (p. 4).

This interpretation also elucidates several analyses of the above-mentioned relations, and suggests important avenues for continuing their investigation, and as Morf (1994) has said, paving the way for 'the acceptance by didactics of constructivist paradigms' (p. 29).

In this paper we will be looking at some of the ways in which genetic epistemology can be applied to and function in mathematics teaching. In order to be more explicit about our intent, here is a very brief excerpt from the text of a lecture given by Guy Brousseau (1991) on *What is at stake in a didactic situation:*

> If I have a didactic phenomenon which is genuinely sensitive to a psychological phenomenon, there must be an object which has meaning in both of them; I must identify the didactic object through which the psyche will express itself in this situation. So I believe that it has a bearing on what is at stake, because ultimately it is what is at stake, the desire which will be the driving force behind all these conversions (p. 149-50).

In the pages that follow, then, we will try to identify certain phenomena or objects in the didactics of mathematics, together with phenomena or objects of cognitive development psychology, the study of which might be enriched by the connection.

The didactics of mathematics: its purpose, objects and methods

To replace the issue of the relations between psychology, education and mathematics teaching, formulated in the 1960s, with the issue of the relations between the didactics of mathematics and genetic epistemology is not an exercise in style. It means first of all acknowledging an important event, the creation of a scientific discipline 'the didactics of mathematics', with its own purpose, objects and methods. It also means asserting that the contributions of genetic epistemology to the didactics of mathematics will be enhanced when they come within the scope of that discipline.

The emergence in France of 'the didactics of mathematics' has had the apparent or visible effect of replacing ideological and often alarmist approaches to mathematics teaching with a scientific analysis of the phenomena associated with it. In his article on 'Syllabuses and didactic transposition', Chevallard (1986) encouraged us to think about the teaching system, the didactic process and syllabuses. He reminds us first that the teaching system is subject to the laws of the didactic process. Acknowledging—and in particular asserting—that it is so has serious implications, setting in motion a search for laws which 'govern the act of teaching and social management' (Chevallard, 1986, p. 32). In addition, it banishes the unrealistic visions or Utopias which might tempt didacticians or researchers hoping to influence teaching. The recent history of set theory teaching in elementary schools shows how the obedience of such teaching to the laws of the didactic process has produced unexpected phenomena and destroyed the dreams of a large number of didacticians and researchers (Conne, 1981).

Only a study of the laws of the didactic process can provide a defence against instant representations thereof which can lead to the syllabus being placed 'in the dock' when the teaching system is considered to be failing. Chevallard (1986) stresses the point: where pupils, teachers and schools cannot be held responsible for the failure, it is the syllabuses that are blamed; and as a result, it is the syllabuses that are changed. Moreover, the more ways are available for assessing teaching, the more we find that syllabuses are being modified on a frequent basis. It is thus the significance given to syllabuses and not the syllabuses themselves which determine their 'true (and tyrannical)' importance. Questioning them is ultimately symptomatic of a representation 'of the teaching system that is a-scientific (pre-scientific?)' (Chevallard, 1986, p. 44).

According to Chevallard, then, mathematics teaching must be brought into 'the modern world of science'. The teaching system must be studied; we must attempt to understand its properties, laws and determinism. That is the purpose of the didactics of mathematics in France. What follows are the subject-matter and methods of the new discipline.

The study of the didactic system governs the study of the interaction between three subsystems: teacher, pupil and knowledge. We will refer principally to

the works of Brousseau (1986*a*, 1990), Mercier (1992) and Rouchier (1991) to define these subsystems and the way they interact. The pupil is the product of a didactic purpose: the person who is taught, the one who is educated. The school is a didactic institution which enables the staff—in their capacity as teachers—to encourage in the pupils—those being taught—the emergence of relations to objects of knowledge. Knowledge is thus placed at the heart of the didactic relation. The pupil's accession to knowledge, and so to the institution, is achieved through situations. It is by and through situations that the pupil will establish a relation to an item of knowledge that is at first external. It is also by and through situations that the teacher effects the re-contextualization and re-personalization of knowledge, emphasizing certain relations to it. Finally, it is by and through situations that the teacher and the pupil inform each other of their own relation to knowledge and of their responsibilities; this exchange takes place under the control of the didactic contract.

Over the last few decades, the didactic situation theory developed by Brousseau (1986*a*) has taken hold among the community of researchers into the didactics of mathematics (and of science) as the pre-eminent instrument for studying the didactic system.

Situation theory 'organizes a reading of didactic events' (Brousseau, 1988, p. 16), in reference to ways in which information and knowledge function. The distinction between information and knowledge is a key element of the theory. Such a distinction also makes it possible to define with greater clarity the foundations and methods of the didactics of mathematics, and to situate the new discipline. In the analyses we will be proposing, we will often refer to this distinction on the basis of the texts published by researchers who have looked at this issue (Brousseau & Centeno, 1991; Conne, 1992; Rouchier, 1991). For the moment we have decided to outline briefly the distinction proposed by Conne (1992).

The distinction between information and knowledge is marked essentially by the subject-situation interaction. Conne (1992) refers to this interaction in the following terms:

> On the one hand, the situation induces information; on the other hand, information enables us to act upon the situation. [. . .] When the subject acknowledges the active role of a piece of information on the situation from a personal standpoint, the inductive link of the situation on that piece of information becomes invertible: he knows. A piece of information thus identified is knowledge (p. 234-35).

What gives teaching its fundamental role regarding the distinction between information and knowledge is its aim of transforming what the pupil knows in a given situation into information that can be used—knowledge—which moreover corresponds to the forms of knowledge to be taught—established knowledge. Established knowledge thus conveys the social practices shaped by an institution.

Distinguishing between information and knowledge facilitates an understanding of the interest and importance of didactic situation theory. In presenting this

theory we shall refer to the research carried out by Brousseau (1981) on teaching decimals.

In his mathematical and epistemological analysis of the concept of the decimal number, Brousseau identifies three phases in the construction of the concept. During the initial phase, the decimal is a proto-mathematical concept (a term borrowed from Chevallard; see Brousseau, 1981); it is used in various methods of measuring and representing quantities. Implicit awareness of the decimal is involved in these methods. At this stage the decimal is recognized neither as an object of study nor as a tool. In the second phase, initiated by combining calculations involving natural numbers and geometric relations and by using decimal notation, the decimal becomes a 'consciously used, recognized and designated tool' (Brousseau, 1981, p. 45); it acquires the status of a para-mathematical concept (Chevallard, 1985; 1991, p. 55). Lastly, during the third phase, the decimal becomes a mathematical concept 'governed by a theory which establishes its definition, properties and epistemological position' (Brousseau, 1981, p. 46).

In didactic situation theory, these different phases in the construction of knowledge enable us to define different types of situation and, hence, different ways in which information and knowledge function. The action situation corresponds to the proto-mathematical phase. The pupil adapts to the situation by constructing an implicit model enabling the problems inherent in it to be solved. Knowledge is thus manifested as the 'individual's way of adapting to the constraints and challenges of the environment and situations' (Rouchier, 1991, p. 33).

In a formulation situation, the pupil is led to explain his model and the information at his disposal and to formulate representations; he interprets his experience in such a way that he can describe to himself and to others the methods and information used to solve the problems inherent in the action situation. Brousseau (1986a, p. 454) wrote in this respect that 'the element of learning situations which justifies such formulation is communication, possibly self-communication'. A game in which pupils compare their models and experiences gives meaning to the formulation. It could be said too that the formulation situation enables the information-tools used in the activity to be identified. This situation simulates a para-mathematical phase in the construction of knowledge.

Lastly, in a validation-institutionalization situation, the pupil is induced to prove, to validate what he is proposing and, eventually, to confer a status on what he has produced, through a process of identification and authentication 'by other knowledge already established and present in a given culture' (Rouchier, 1991, p. 35). It is through such a situation that knowledge becomes object; the situation corresponds then to the mathematical phase of the construction of knowledge.

Situation theory, as it models the functioning of information and knowledge, is a basic tool for studying the didactic system. That is why the problem of the devolution of a-didactic situations or the pupil's access to a learning situation (when the teacher's goal comes second to the pupil's responsibility) has been

important since the beginning of studies on the didactics of mathematics. It is also in reference to situation theory that the process of institutionalizing knowledge has been examined. Lastly, the methodology of didactic engineering has been created as an application of situation theory in order to 'test the theoretical constructions developed in research by involving them in a production mechanism' (Artigue, 1990, p. 285) and to 'take into account the complexity of the class' (Douady, 1987, p. 222).

The standpoint of genetic epistemology and its influence on the didactics of mathematics

As we have just shown, the didactics of mathematics is a discipline defined by a specific purpose, objects and methods. It deals with 'already established knowledge, that is knowledge which has its place in a given society and which is passed on through a programme taking the form of teaching' (Rouchier, 1991, p. 36). This established knowledge is a structuring object in research in this field. Understanding the transmission of this knowledge in teaching is a fundamental aim. It is an aim alien to genetic epistemology, although this does not mean that Piagetian theories can shed no light on understanding teaching. This is how Piaget referred to the relations between psychology and mathematics teaching:

> Mathematics teaching depends to a large extent on the idea one has of mathematics and consequently its epistemology. While a mathematician would naturally never consult a psychologist to find out how to demonstrate a theorem, [. . .] the issue of the foundations of mathematics is completely different [. . .]. From the epistemological point of view, current trends in mathematics point towards a clearly constructivist form of structuralism (Piaget, 1970*a*, p. 227-28).

Piagetian studies aim to understand knowledge, its nature and its growth. This project is expressed in many ways in Piagetian works:

> This discipline [psychology] has set itself the task of interpreting science as the product of human mental activity, or, and it comes to the same thing, of explaining how human thought itself can produce science as a coherent system of objective knowledge (Beth and Piaget, 1961, p. 325). The specific feature of genetic epistemology is thus to seek to identify the roots of various types of knowledge in their most elementary forms and to follow their development to the highest levels up to and including scientific thought. [. . .] The specific problem of genetic epistemology is, indeed, the increase of knowledge, the transition from knowledge that is less good or poorer, to a knowledge richer in understanding and range (Piaget, 1970*b*, p. 6-8).

This project is at the basis of studies carried out on the construction of the operations and structures of intelligence. Such constructions define both the act of knowing and the product of that act, the subject-object relation and the product of that relation in the subject (Morf, 1994).

It is this cognizant subject who enters the didactic system and whose encounter with the object or environment is organized by the didactic situation, which is itself organized by established knowledge. Understanding how this set-up affects the behaviour of the pupil, a cognizant subject, seems to us to be fundamental in locating possible relations between the didactics of mathematics and genetic epistemology.

Established knowledge and didactic functioning: respective spheres of the didactic subject and the epistemic subject

The placing of the subject in relation to an established situation which is both structured and structuring is an old question, whose introduction into didactic inquiry constitutes an important development for the latter (Rouchier, 1991, p. 28). In what follows we would like to set in train consideration of the problems posed for genetic epistemology by didactic functioning as a result of its essential, even existential relation to established knowledge. We would also like to show how psychological inquiry can elucidate didactic functioning.

ESTABLISHED KNOWLEDGE IS IDENTIFIABLE, VISIBLE, RECOGNIZABLE, USABLE AND TEACHABLE

Established knowledge is thus presented in school textbooks, curricula, didactic works, research publications and the mathematics class. From his earliest work in didactics, Chevallard (1982; 1985, 1991 edition) has taken an interest in knowledge. He defines it as follows in one of his publications (1988):

> An area, or to be more precise a body of knowledge, is in fact a system with an objective existence, which corresponds to the organization of various types of semiotic content (discursive, practical, etc.), which existed before the specific individual who encounters it as the subject of a given institution in specific circumstances, that is through given situations, under the jurisdiction of a given contract, in an institution where he himself appears on particular terms. [. . .] We can then say that the individual has a (certain) relation to knowledge (p. 17).

In his work on the anthropology of knowledge (1991), Chevallard proposed a definition of a mathematical object which sheds light on the issue of the relation to mathematical knowledge:

> An object (for instance, a mathematical object) is something that emerges from a system of practices in which material objects are manipulated, which are expressed in different semiotic registers: the oral register of spoken words and expressions; the register of gesture; that which is written or drawn (graphics, formalism, calculations, and so on), that is the written register. And we can add to these three registers the presence of some material objects (in the usual sense of the term). [. . .] We shall then call these objects, as taken in

practices, praxemes. An object thus emerges from a system of praxemes. It emerges in practices (p. 110).

It is certain praxemes which are manipulated in mathematical activity. This statement is most instructive for the didactics of mathematics, since it renders 'unintelligible and inoperative' any approach which bears on the construction or understanding of knowledge by a pupil. Hence, declaring that a pupil does not understand, for example, fractions when that pupil has actually been taught fractions means nothing in itself for someone who does not know which procedures the child has been shown. The statement only has meaning for the person who pronounces it. Some praxemes are thus prioritized in mathematics syllabuses and textbooks. These praxemes can represent or indeed present the object and designate it. For example, the object 'subtraction of whole numbers' can be named whilst the written procedures and their accompanying commentary are effected. Anyone asking pupils to explain how they carry out a subtraction can recognize expressions that they all use such as 'I borrow a ten from the tens column'; the pupils have not only remembered the changes that are needed, but they can also name those transformations.

A number of semiotic objects are thus created in mathematics: schemas, written notations and so forth. These objects are integrated into new practices. What mathematics texts show is the result of the normalization of these praxemes, retaining only standard objects. Mathematical knowledge thus appears purged of the history of the praxemes from which it has emerged.

Teaching must prepare this knowledge so that pupils can 'digest' it. In so doing, it adds certain praxemes which might be very different from the praxemes which have gone to make up such knowledge (various semiotic objects). It also 'peppers' teaching with objects of everyday language, which, as Laborde (1982) has shown, can add to pupils' confusion.

This is how didactic transposition takes place (Chevallard, 1985, 1991; Conne, 1981), and it is inevitable whenever teaching is required. What becomes of the relation to knowledge, established knowledge? For the pupil, it will be the relation constructed on the basis of the different semiotic objects which have been used in his/her practices and then instituted. For the teacher, it might only be realizable through the praxemes of his professional milieu (the class). It might also be a combination of those praxemes and the praxemes of his mathematical activity and history. Making the connection between the relations to knowledge of the teacher and of the pupil is one of the most demanding and complex activities of teaching; it takes place in the situation. It is in this context that Rouchier (1991) talks of the institutional functioning of the situation: 'It is in fact essential that this situation institutes the target information, or at least the material and/or symbolic practices which prove that they are at the heart of the relation the pupil has with the situation. In this respect, the situation is an institution' (p. 36).

The process of institutionalization 'officializes' in this way the relation to knowledge, legitimizes it and gives it the status of knowledge. This relation can be

given a designation and a name. It is the didactic subject's relation to knowledge that is institutionalized, and this relation reveals itself in practices, which can be extremely straightforward. For instance, it is not difficult for an eleven- or twelve-year old pupil to add fractions with the same denominator, as long as the total is an irreducible fraction, less than one (e.g.: 1/9 + 3/9 + 4/9 → 1 + 3 + 4 = 8; 8/9); but institutionalizing the procedure used in relation to the addition of fractions is questionable.

The relation attributed to the didactic subject can also be confused with that of the teacher; very often, without knowing it, the teacher guides the pupil's solutions, formulates them and comments on them. These didactic effects are remarkably well described by Brousseau (1986*a*, 1986*b*). It is equally possible that the psychological subjects do not believe they know or have learnt 'anything'; they were able to carry out 'successful' actions but not able to interpret or validate them. If, according to Piagetian theory, the schema is the functioning unit in problem-solving, the 'plan of actions which might be repeated actively' (Inhelder et al., 1992), its functioning is also heuristic, even opportunistic; it enables a situation to 'speak', and problems to be solved. We might be able to solve problems, but that does not mean that we recognize what makes our actions relevant and necessary. Inhelder and de Caprona (Inhelder et al., 1992) make a distinction between procedural schemas and re-presentative schemas which is highly significant for cognitive psychology and might also help interpret certain relations to established knowledge:

> Re-presentative schemas are opero-semiotic; they apply operations to symbols or signifiers rather than to objects, and have an inferential function which includes practical applications (anticipating; planning; reconstituting) and theoretical ones (modelling; deducing; explaining). [. . .] On the other hand, procedural schemas are series of actions used to reach an end: they are difficult to remove from their context and their conservation is limited since a means to an end is of no further use when the subject moves on to the next means (p. 41-42).

The relation to knowledge which is instituted in the didactic relation allows the information which defines it to enter into the representation of other situations. The identification of this relation with the organization of various types of semiotic content enables it to assimilate or accommodate. No reading of pupil behaviour can, we believe, disregard this capacity. Studies by Brun et al. (1993) on errors in calculations show that the way division is set out can determine what the pupils do and lead to surprising accommodations where there is inadequate numerical and numeral control. This work of accommodation also ensures the functioning of schemas, such as the 'sharing-distributing' schema; the Piagetian notion of schema is thus shown to be valuable in interpreting the relation of these pupils to 'division'. Such relations do show the functioning of a didactic subject-who is also a psychological subject; we will have the opportunity later to demonstrate the relevance of studying how these subjects function in the analysis

of didactic events. These studies show us too how established knowledge can be taught. If it had no semiotic substance, it would be hard to envisage teaching it.

In our teaching work we have also been able on many occasions to experience the assimilating and accommodating functions of established knowledge. In some of our practical work we have also been able to show how the epistemic subject and the didactic subject 'co-operate'. Thus, while looking at the teaching of multiplication with trainee teachers, in order to determine which of the possible multiplication techniques the students know, and in order to analyse the information and skills which determine and control the calculations that are made, we usually give them, without comment, the diagram given in Figure 1, which corresponds to the application of the technique known as 'Russian peasant multiplication'.

FIGURE 1: 'Russian peasant multiplication'

	342	1,083
→	171	2,166
→	85	4,332
	42	8,664
→	21	17,328
	10	34,656
→	5	69,312
	2	138,624
→	1	277,248
		370,386

The students have first to identify the skill it represents and to explain the diagram. Of course, none of them can do so spontaneously. They can, however, point out how different numbers are related through multiplication. As soon as we tell them that it is a multiplication problem, the overwhelming majority of students can explain the diagram. They very often proceed in this way: (a) multiplication of 342 and 1,083 in the usual way leads to recognition of the result 370,386 as the product; (b) adding the numbers in the rows marked →; the number 370,386 is once again obtained; (c) multiplication of numbers in the rows marked →; comparison of these products to the expected total, which is done by calculating the differences and leads to the realization that the difference between 85 x 4,332 and 171 x 2,166 is indeed 2,166; (d) search for an explanation of this result by examining the relations through multiplication between the numbers in various columns, by setting out each of the multiplications in the usual way; it is then that several students observe that multiplication and division by the same number cancel each other out, as long as the remainder of the division is nil.

The identification of a functional invariant, marking according to Piaget (1966) 'the agreement of thought with things and the agreement of thought with

itself' is indeed a product of the epistemic subject, or at least, in the situation we are describing, a product of the co-operation between the cognizant subject and the didactic subject. This product will then determine the students' relation to other objects of knowledge, for instance the multiplication and division of decimal numbers. Indeed, in the course of this work we ask the students to explain the procedure for multiplying and dividing decimal numbers (e.g. in division the removal of the decimal point, adding 0 and so on). Their explanations are based then on the functional invariant constructed in the initial situation.

These last results show us that multiplication and division were established knowledge in the education received by these students, that such knowledge contains information which enters into the representation of the situation, that it has a name which effects the union of the signifier (e.g. the word multiplication) and the signified (Barthes, 1985), and that the signified can be seen as a conceptual field according to the definition given by Vergnaud (1991). This inclusion in a conceptual field as shown by our interpretation of the students' behaviour activates—it is difficult to find a more suitable expression—the psychological and didactic subject.

What is the didactic relevance of these results obtained in a very specific setting, that of teacher training? We are inclined to think that the results inform us too about the didactic process, and, in particular, the conversion of information into knowledge, since the situation we have looked at can be seen as a didactic situation. Multiplication and division are established knowledge governing the creation of the situation. Its representation indeed brings into play information which transforms it; the reason for such transformations can be sought in the information which becomes an object of thought in the process of formulation and validation. We could say that knowledge appears 'as production', in the meaning Rouchier (1991) gives to the term. This knowledge can also be transferred to another situation and, retaining its essential character as knowledge, acts on the new situation, transforming it. Work on the new situation affects the significance of the knowledge since its meaning is linked to a situation or a practice. These interpretations are based on analyses by Conne (1992) and Rouchier (1991) of the processes whereby information is converted into knowledge and knowledge is transposed.

Lastly, we believe that in this situation the role of the teacher with a model of established knowledge, a model of the didactic subject and a model of the epistemic subject must not be ignored. It is this information which enables the teacher to read the students' behaviour and propose a second situation.

ESTABLISHED KNOWLEDGE IS AN ORGANIZER OF THE COGNIZANT SUBJECT IN A DIDACTIC SUBJECT

If, in a didactic situation, a subject's encounter with a specific context is indeed an act of knowledge and if the product of that act resides in the subject, a supposition which acknowledges the assimilative and accommodating nature of the

cognitive structures defined in Piagetian theory, then the product must take place in the register of established knowledge, otherwise it loses all didactic significance. It is this transfer which gives such knowledge a role in organizing the cognizant (epistemic and didactic) subject in a didactic relation. In this connection, Brousseau (1988) writes: 'The *official* acknowledgement by the pupil of the object of information, and by the teacher of the pupil's learning is a very important social phenomenon and an essential phase in the didactic process; this dual recognition is the object of INSTITUTIONALIZATION' (p. 17).

The reference to established knowledge in the didactic process binds pupil and teacher in a didactic contract unlike the experimental contract linking subject and psychologist in psychological research. Studies by Schubauer-Leoni (Schubauer-Leoni & Grossen, 1993; Schubauer-Leoni & Ntamakiliro, 1994) have identified differences between pupils' answers according to where they were given (in or out of class), the mathematical content of the problems, and the private or public nature of the answers.

In a study that was carried out (Lemoyne & Gauthier, in press) on how pupils related to mathematical formulae used in chemistry teaching, which we investigated after teaching that discipline, we demonstrated how reactions differ according to whether the formulae are introduced in a mathematics class or a chemistry class. Thus, for eleven of the twenty-eight pupils questioned in chemistry classes, the formula p1v1/t1 = p2v2/t2 contained two calculations to be done and indicated that the results of the calculations must be equal; these pupils recognized too that the formula was linked to the laws of gases. In mathematics classes, only a few interpretations of this kind were proffered (three pupils out of twenty-five). Seventeen of the twenty-five pupils in these classes, compared to nine out of twenty-eight in chemistry classes, linked the formula to fractions, ratios and proportions and assigned numerical values to the letters to demonstrate by the calculation what they understood by equal ratios. Released from the constraint of using the formula in chemistry, but at the same time eager to show what they knew about mathematics, these pupils showed that they were better able to see the formula in mathematical terms.

As Brun and Conne have stressed (1990), the pupil's functioning cannot be attributed to the cognitive process alone; it must also be examined in the light of the situation. When pupils operate in a didactic situation, the need to 'show the teacher' what they have learned or know strongly influences what they produce.

If a pupil's showing is institutionalized or recognized as satisfactory with regard to established knowledge, then that result will still be discernible when a specific concept is examined several years after it has been taught. For instance, we have shown how, for most pupils and teachers (Lemoyne, 1994), the skill of 'writing fractions using a common denominator' organizes their conduct in solving a problem which involves putting fractions into order (e.g. 11/12; 24/100; 7/8; 3/4; 3/10; 16/36; 1/4) several years after being taught the skill. The institutionalization of this skill in education seems to place these individuals immediately in a didactic situation where they have to show what they know,

what they have learned. In fact several of them proved capable of carrying out the task by using what they knew about fractions, when we described the task differently (e.g. 11/12 is bigger than 7/8 because you need 1/12 to make 11/12 up to 1, and 1/8 to make 7/8 up to 1 and because there is less missing from 11/12 than from 7/8); the comments and justifications accompanying the procedure demonstrated a satisfactory approach to fractions. However, these people did not accord the same status to the second solution as to the first. The conventional, established solution was judged to be better than the other even though it involves numerous multiplications and a rather significant increase in the common denominator. A good many people were also unable to explain the procedure that they had employed.

The move from didactic to psychological research is not a straightforward one. In the case of fractions, as elsewhere, the introduction of the tests designed by Noelting (Noelting, 1980; Noelting & Béland, 1988) to examine the development of proportional reasoning in mathematics classes led to their transformation for didactic purposes, such that research which was supposed to be psychological focused in reality on relations to a new object of established knowledge.

Moreover, we have here a fundamental didactic problem. We cannot, from an examination of a pupil or other individual's behaviour in a process of investigating or evaluating mathematical knowledge which has been taught, draw conclusions about the state of knowledge or information, or indeed about the pupil's information or knowledge, as we might believe a psychologist does when looking at how cognition functions. At most we can talk about relations to objects of taught or established knowledge. Obscuring this distinction between the psychological and the didactic, between the functioning of cognition and the functioning of knowledge in the institution that is the school, several researchers in mathematics teaching have focused on the inadequate and mistaken beliefs of pupils and teachers and derived from those beliefs proposals for a new teaching, or a re-teaching, which might lead to the construction of 'real knowledge' or 'real information'. Several of these proposals consisted moreover of elevating the information brought to light in various investigative situations to the status of knowledge to be taught. In the case of the study on fractions, a proposal of this type would be that pupils should be taught how to order fractions using the findings from the task version designed for the psychological examination. There is no need to stress the psychological and didactic naïveté of such a proposal.

Does this mean that methods of psychological research into taught and learnt knowledge have no relevance for the didactics of mathematics? It does not seem to me to be an appropriate way of putting the question since, in the study we are recounting, the initial reactions of the pupils and teachers show that they focused on the product of actions. As we understand Conne's studies (1992; Brun et al., 1993) on the conversion of information into knowledge, these reactions are governed by a skill whose utility is measured by its result, the product; the success of the task—success measured by the fact of having produced a reply in an acceptable form, or more generally, having made some attempt at

the task—thus constitutes a criterion for the utility, or even the maintenance, of the skill.

We could complement this analysis of skill by recalling some conclusions of Piagetian studies on the transition from success to understanding. In his book *Réussir et comprendre* (1974), Piaget wrote:

> To succeed is to understand in action a given situation to a degree that is sufficient to reach the goals that have been set, and to understand is to manage to dominate in thought the same situations until you can solve the problems they raise as to the how and why of the links observed and used in the action (p. 237).

Raising the problem of the functioning of knowledge in teaching is therefore to acknowledge the weight of established knowledge on the cognizant subject in a didactic situation. It also means acknowledging the problems raised by any attempt to alter the relations of pupils to knowledge which has been taught. Several teachers and researchers believe that it is enough to repeat the situations to modify the relations of pupils to knowledge. That means behaving 'as if' we could endlessly replay action situations or a-didactic situations. In our study on secondary pupils' relations to numerical and algebraic formulae (Lemoyne & Blouin, 1994), we were confronted with this desire to transform the relations of many pupils to such formulae. The pupils (A-group pupils) knew how to solve simple algebraic equations but were clearly unable to explain the changes they had made in the formulae.

We therefore decided to present to other pupils who had not yet been taught algebra (B-group pupils) the following equations and ask them to find the values of x and c: 1) $4x + 4c = 40$; 2) $6x + 2c = 48$. Several of the pupils found relations between x and c, on the assumption that the same number was assigned to the letters and by testing that assumption; thus they found that x was 4 more than c. They all then discovered the values of x and c; some of them proceeded as follows: $40 - 16 = 24$; $24/8 = 3$; $x = 7$; $c = 3$.

We presented this solution, the product of the epistemic and didactic subject, to the A-group pupils and asked them to evaluate it and to demonstrate how the operations they carry out when they solve such equations might reflect the B pupils' solution; we suggested to them a new situation, different from the one they had been given previously. Several of these pupils then managed to explain the relationship between their solutions and the B-group pupils' solution.

In presenting this way of proceeding, we by no means intend to imply that it is exemplary. We want only to bring out a few characteristics: (a) the situation gives the information used by the A-group pupils the status of knowledge, it recognizes that they have learned algebra; (b) the situation is a new one for A-group pupils and its aim is thus to transform their relations to knowledge about algebra by making them perform in another register or on another level, or even introducing them into a tool-object dialectic (Douady, 1984). Lastly, we have shown that the didactics of mathematics cannot ignore the problems of the conception and

management of didactic situations for the transformation of knowledge that has already been taught; in the course of transformation, the knowledge which is 'already there' (Rouchier, 1991; Mercier, 1992) fills, we could say, exaggerating slightly, almost all the didactic space.

The conception and management of situations for mathematics teaching

In the previous section we showed the weight of established knowledge in the didactic process and in the creation, operation and implications of didactic situations. Highlighting relations to the objects of this knowledge is a starting-point for looking at didactic situations, for attempting to reconstitute the academic history of these products of teaching and, finally, for exercising more effective control over didactic situations. Thus, although didactic engineering is based on a mathematical and epistemological analysis of knowledge, it does not neglect the prior analyses which describe the construction of the major categories of information which are linked to it and, lastly, it situates this knowledge in relation to the choices made and contents of the teaching which has been carried out.

In the following text, we would like to look at *the possible links between the didactics of mathematics and genetic epistemology or Piagetian psychology,* in the light of the conception and management of situations for mathematics teaching.

All situations bring pupils into a relationship with a certain environment, confronting them with one or more problems. In order to bring out the relation or relations of a pupil to knowledge, it is not enough to make him or her solve problems related to that knowledge. Descartes' declaration, which Boirel used as the epigraph for his book *Comment résoudre aisément des problèmes de mathématiques* [How to solve mathematical problems easily] and Boirel's own observations on the difference between a good pupil and a more brilliant one, are still food for thought today. 'We will never become mathematicians [. . .] though our memory holds all the demonstrations made by others, if our mind is incapable of solving all sorts of problems. [. . .] Then, in fact, we would appear to have learnt, not science, but history.' (Descartes, *Rules for the direction of the mind, III,* quoted by Boirel, 1964, p. 10).

So, do you understand the psychological difference which separates you, you, the good pupil who understands the mathematics lesson, from your class-mate who solves the problems easily? Whilst for you algebraic figures and expressions are static, defined in an immutable way by the wording, for your class-mate on the other hand they appear form the outset in dynamic form: they are seen as bundles of functional possibilities, ready for immediate mobilization according to the problem in question (Boirel, 1964, p. 13-14).

Research in the didactics of mathematics has shown how the conception of problems which might come up in didactic situations is closely linked to a

mathematical and epistemological analysis of the knowledge to be taught. Research into fundamental situations with a view to teaching knowledge is therefore of increasing interest to researchers in didactics. Brousseau (see Berthelot & Salin, 1992) defines the project in this way: 'To model the functions of information through non-didactic situations representative of the practices of reference institutions and to use those models to analyse teaching situations, and for engineering and monitoring' (p. 34).

Brousseau's work (1981, 1987) on decimals teaching and that of Berthelot and Salin (1992) on space and geometry teaching show that the choice of fundamental situations can lead to the construction of didactic situations in which the emergence of relations to mathematical knowledge can be better controlled. The construction of such relations is also regulated by the epistemic subject and need not be completely under the sway of the situation.

Blouin (Blouin, 1993; Lemoyne & Blouin, 1994) has adapted some of the situations proposed by Brousseau for decimals teaching to use in teaching rational numbers to pupils with difficulties in learning mathematics. Her analysis of these pupils' behaviour shows how the functioning of the addition schemas for natural numbers, by accommodation and co-ordination with other schemas such as those for sharing out, putting together and measuring, can result in behaviour very unlike that which might be expected from an independent construction of decimals and naturals or a construction of decimals through Q.

For example, in order to find the multiplier which, if applied to 14.6 (one of the measurements of a sailing boat) gives 21.9 (the same measurement of a second sailing boat), a pupil finds the difference between the two measurements, namely 7.3, then transfers that difference to the measurement 14.6 and concludes that the second sailing boat is 'one and a half times' the length of the first; the pupil then proposes to do first + 7.3, then x 1/2. This interpretation is indeed an additive transformation (Vergnaud, 1981, 1983); the difference referred to is one that can be interpreted as a fraction of the initial measurement, which leads to the unit being left out of the transformation according to the formula 'one and a half times'. The vitality of an additive model has been pointed out on many occasions in Piagetian studies on the concept of function (Ricco, 1982; Piaget et al., 1968). Must we take it into account when planning didactic situations? If so, how?

If the genesis of additive and multiplicative structures is a research topic in cognitive development psychology, then learning by pupils of arithmetic procedures is a teaching problem. Mathematical and epistemological analyses shed little light on teaching this area of knowledge, which is really only of practical interest to didacticians. In the search for situations in which to teach it, psycho-cognitive or psycho-genetic analysis coupled with a mathematical analysis can lead to a choice of situations not envisaged in the light of a mathematical and epistemological analysis. This is how acknowledgement of pupils' cognitive structures and numerical schemas led Vergnaud (Vergnaud et al., 1979; Vergnaud, 1981, 1983) to define classes of addition and multiplication problems which

direct the teaching of these operations. The finding is important for didactics, since there are now countless research projects based on this distinction. The importance of the finding must also be acknowledged for the theory of conceptual fields, also developed by Vergnaud (1991).

According to this theory, concepts develop in connection with situations or problems that learners know how to solve, and with others for which they seek solutions. The properties and invariants which make up the concepts constructed by learners depend on the nature and characteristics of the situations. Diversifying the situations enables the various meanings of a concept to be addressed; this diversity facilitates the abstraction of the essential properties, or the invariants of a concept. The notion of theorem-in-action stems from the concept of invariants.

The work done by Brun and Conne (Brun et al., 1993, 1994) on mistakes in written division is heavily influenced by the conceptual fields theory. These researchers have shown that pupils have 'several ways of organizing the algorithm-schema under construction', that 'the graphic layout of the operation takes over and even organizes the action and also masks, by the same token, the meaning of the series of actions needed to execute the operation'; that the basic sharing-distributing schema 'serves to assimilate all division situations' and, lastly, that the 'calculation situations proposed to the pupils provide, by their variety, various kinds of resistance to the assimilating schema: that is the point at which various mistakes associated with the characteristics of situations and ways of accommodating the schema to the situations, are made' (Brun et al., 1994, p. 123).

Brun and Conne suggest designing didactic situations which present 'the algorithm as a curiosity to be explored by means of the numerical and numeral information at our disposal, until we can find the key to it' (Brun et al., 1994, p. 130). Finally, let us note that, in an extension of these studies, the didactic treatment of mistakes has been examined in a piece of original research carried out by Portugais (Portugais, 1992, 1995 ed.; Portugais & Brun, 1994) on students in a teacher-training course. He describes twenty-six forms of strategy for the didactic treatment of mistakes among the teachers-to-be during the didactic sequences they carried out. He also shows how the position of these students 'in a didactic double-bind', as both trainees and teachers, increased the complexity of the analytic work required by the didactic process; the findings of Leutenegger's study (1994) also show that it is impossible not to look at the interaction of roles when analysing the didactic sequences prepared by the student 'teachers-to-be'. Taken together, these works open up an important area of research in the didactics of mathematics.

In a study of multiplication teaching (Vincent, 1992; Lemoyne et al., 1993) given to pupils between 8- and 10- years-old, we wanted to test the assimilating and accommodating force of schemas in didactic situations. An analysis in terms of schemas of additive and multiplicative relations (e.g. more, times as much, times more, and so on), coupled with an analysis of numerical information about

multiplication and addition, led to the development of teaching situations which provided an opportunity to contrast numerical schemas and information and co-ordinate the information. We have thus described three main moments when pupils formulate relations of multiplication.

In an initial period in which addition and multiplication are not differentiated, schemas of equivalence and the iteration of units hold the key to the solution of multiplication problems. The problem 'Mary bought twice as many fish as Julie. In all they bought 15 fish' is solved as follows: two sets of six fish are formed, in line with an equivalence schema (the same as); three fish are then added to one of the sets, in line with an 'add *n*' schema. In a second stage, the conditions for the application of the schemas are extended. The statement of equivalence between the sets can be based on a comparison of the elements or sub-sets (parts) making up the sets; the iteration schema can then function through the bringing together of elements, on the basis of parts made up of elements. Multiplication viewed as repeated addition thus characterizes the '*n* times more' schema. In order to illustrate the preceding problem, the following steps can be carried out: (a) produce two identical sets of four elements; (b) add to one of the sets two sets of the same size as the others, thus modifying the total number of elements in order to make the schemas work (the total number of fish is no longer 15 but 16). Lastly, in a third stage, multiplication and repeated addition are differentiated; the co-ordination of the previous schemas stems from the construction of functional invariants in those schemas, from the co-ordination of the relations between elements, parts and whole, and from varied numerical information. For the previous problem, the equivalence schema 'same as' is co-ordinated with the schema 'do once'; 'do twice' thus becomes 'do the same twice'. This accommodation of schemas takes into account numerical information.

In this study, the interpretation of pupils' behaviour in terms of schemas enabled us to set up situations aimed at transforming relations to multiplication. It also disclosed to us all the complexity of the process of interaction with regard to information within one class; so, the fact that we were able to interpret the comparable behaviour of pupils in terms of schemas does not mean that for the pupils concerned the information contained the same significance and could be recognized, explained and accommodated in encounters with other situations. In order to help us understand better, let us recall what Piaget (1972) said about becoming aware:

> Common sense provides a completely inadequate (not to say wrong) idea of awareness representing it as a sort of enlightenment which illuminates realities until then obscure, but changes nothing else. [. . .] In fact, becoming aware is much more than that, since it consists of transferring some elements from a lower, subconscious level to a higher conscious level. [. . .] Becoming aware is therefore a reconstruction on the higher level of what is already organized, albeit in a different way, on the lower level [. . .] the subconscious is furnished with sensory motor or operational schemas already organized in structures, but expressing what the subject can 'do' and not what he or she thinks. [. . .]

> This process of cognitive awareness [. . .] is completely different from mere enlightenment [. . .]: it is a process of reintegration and a resolution of conflicts thanks to a new form of organization (p. 16-18).

We have thus been obliged to experience the didactic-epistemic dialectic: (a) didactic: teaching solutions to multiplication problems, taking the risk described by Morf (1994) of creating closed packages of information which are nevertheless economic in the short term; (b) epistemic: promoting without teaching solutions—the growth of potential of a generative type (information seen as potential for action) which does not produce the performance expected at the outset. As Vergnaud stressed during a round table on curricula (see Michel-Pajus, 1986):

> The formation of a concept takes years, but that formation has to be managed in the short term of teaching situations. Furthermore, we observe very significant intervals between children. We must look at their past but also into the future, for the reinforcement of a concept, in elementary school for instance, can be an obstacle to the subsequent acquisition of further information (p. 54).

In brief, the problem raised by the institutionalization of information seemed to us still more delicate than we might have expected on the basis of didactic studies. We finally wondered about the didactic relevance of this knowledge about multiplication which, in practice, is never, as far as we know, the subject of teaching even though it enters the composition of several taught subjects. We are touching here, we believe, on problems relating to the boundaries between the purposes and subject-matter of psychology and didactics.

Conclusion

Didactics, with its aim of understanding the didactic system, not only reflects a desire to establish a discipline distinct from cognitive psychology. It represents a clear statement of the fact that the pupil's functioning as a cognitive system can only be studied by taking account of the educational design embodied in a store of knowledge already present, which lies at the heart of the didactic relation. In order to understand the functioning of this cognitive system, it is therefore not possible to disregard such knowledge. It is, we think, on this condition that genetic epistemology might elucidate certain didactic facts.

Observing this condition, we would like to return to the thorny problem of objects and control of didactic situations, a problem we have raised on several occasions in our work and which involves the whole process of converting information into knowledge, the whole process of institutionalization. The studies carried out by Conne (1992) and Rouchier (1991) propose analysis of this process and bring up fundamental questions. In situation theory and in his analysis of teaching decimals, Brousseau (1981, 1987) shows how articulation and valida-

tion situations could be played out. Analysing the interactions of information in such games might shed light on how the process functions.

Recent work on cognitive microgenesis might provide concepts and tools for examining this process. Thus the distinction between procedural schemas and re-presentative schemas (Inhelder et al., 1992) deserves, I believe, to be retained, amplified and tested in the analysis of information interaction. These schemas show the different ways in which information functions. The analyses by Saada-Robert (1989) and Blanchet (Inhelder et al., 1992) of the representation of problems show how a subject's experience in problem-solving can be read in different ways, from recording what has in fact been done, to schematizing the causal and teleonomic aspects of the cognitive experience. The first method is based on procedural schemas, and the second on re-presentative schemas. If the knowledge is useful information (Conne, 1992), the conversion of information to knowledge is difficult to envisage without the functioning of re-presentative schemas. However, having recourse to such schemas is not enough to describe the process. Saada-Robert's studies (1989) allow us to glimpse the sheer complexity of the process. She stresses the need to envisage microgenesis in terms of two aspects:

> changes in meaning concerning the practical or conceptual schemas used (linked to previously-formed models) and concerning objects (real or imagined including their relations); and transformations of control, concerning the organization of actions and meanings according to the aim (p. 195).

The findings of these psycho-genetic studies, together with cognitive hypotheses made in didactic studies, lead us to focus didactic analysis of the information-knowledge conversion process on analysis of the process whereby the skills, thought-out knowledge and established knowledge defined by Conne (1992) are elaborated. The didactic analysis initiated by Brun and Conne (1990) of protocols for observing information interaction in ongoing situations appears to be a promising route for studying this process; it might also assist better definition and characterization of the articulation, validation and institutionalization situations which play a decisive role in the process.

Notes

1. The author wishes to express her gratitude to Jean Brun and François Conne. The completion of this study has benefited enormously from the joint ventures the author has been engaged in for several years with these researchers. Their research into mistakes in division has enabled the full importance of the concept of schema and the theory of conceptual fields in the analysis of didactic protocols to be grasped. Jean Brun's study on the relationship between cognitive psychology and didactics and François Conne's study of the information-knowledge conversion process have also been essential sources of inspiration in analysing links between the didactics of

mathematics and genetic epistemology. The author would also like to thank Jean Portugais for his most valuable comments on the analyses carried out in this text and for the published research he placed at my disposal; lastly, thanks are due to Jacinthe Giroux and Sophie René de Cotret for reading and commenting on the first draft of the text.

2. This sentence used as an epigraph can be read in context in Piaget, 1969, p. 48.

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SOME OF PIAGET'S FUNDAMENTAL IDEAS CONCERNING DIDACTICS

Gérard Vergnaud

Piaget was not interested in didactics, yet didacticians are very interested in Piaget. Is this a misunderstanding on their part or an error of judgement on the part of Piaget? Could critical analysis in the light of research findings available today in the field of didactics give us a deeper understanding of Piaget's scientific contribution to educational research?

The centenary of Piaget's birth is also the centenary of Vygotsky's birth. It is a strange coincidence that the two psychologists who have made the most outstanding contribution to the psychology of advanced cognitive functioning and educational psychology during this century should have been born in the same year, should never have met and should often be regarded as leading two contrasting if not contradictory schools of thought. Or do their approaches in fact converge and complement each other rather than conflict?

These are a few of the questions that I should like to address in this article.

Knowledge as a process of adaptation

This is probably Piaget's most fundamental idea and it runs through his whole work. It clearly owes something to the fact that Piaget started out as a biologist, deeply influenced by the theories of evolution—those of Lamarck more than Darwin's. What Piaget did was to transpose this fundamental idea of adaptational development from the evolution of species to the development of the child

Gérard Vergnaud (France)
Director of Research at the Centre national de la recherche scientifique, Paris. For fifteen years he directed the research group 'Didactique', a national network of researchers on the teaching of mathematics and physics. He studied under Piaget and he showed an early interest in research on mathematics teaching and learning, on which subject he has written many articles and a book entitled *L'enfant, la mathématique et la réalité* (1981), which has been translated into several languages. He has also studied the analysis of professional skills in adults and of the ways in which these skills are acquired.

and more specifically of the child's thinking processes. He made a vigorous case for his view that, to understand knowledge, one must study the way in which it develops. In other words the idea of the adaptational evolution of knowledge in the child enabled him to embark on a scientific project which consisted not only of developing and justifying the thesis that a subject's current knowledge is the result of the interaction between experience and previous knowledge (i.e. the interaction thesis) but also the thesis that knowledge results mainly from action on the environment, since it is principally through their actions that subjects test and modify their knowledge (the operatory thesis).[1]

It was this idea that didactics inherited and put to profitable use. Indeed the chief merit of didactics is to have focused on creating situations likely to stimulate the development of pupils' activity and knowledge through a process of adaptation. Piaget did not see (or did not wish to see) this direct consequence of the interactionist and operatory thesis, namely the possibility of guiding the learning process by a judicious choice of learning situations. Didacticians, on the other hand, particularly in the French-speaking world, were well aware of this possibility. It was Brousseau, a specialist in the didactics of mathematics, who expressed this idea in its most systematic form. He draws a distinction (over-emphatically perhaps but usefully) between (a) situations involving action, in which the aim is to perform an action successfully; (b) situations involving the formulation of ideas, aimed at producing a message and communicating; and (c) situations involving validation, in which one must show that a particular sentence or theory is valid and enlist support for it. In all three types of scenario adaptational processes are at work—first and foremost in action, which is both the source and the criterion of operatory knowledge.

However, the choice of situation does not depend solely or even mainly on psychology. The epistemology of the subject taught is a key criterion, as epistemology clearly raises the question of the links between knowledge and the practical and theoretical problems to which that knowledge can respond. Some may regard that as a rather restricted theory of knowledge but it is fundamental for an epistemology of concepts and techniques: to *what practical or theoretical problems does the introduction of a particular concept, or a particular property of a concept, or the discovery of a particular technique or procedure correspond?*

Psychology then comes into play by setting this fundamental link between problems and knowledge in the context of each of the stages through which the students can progress. This will determine which key theoretical and practical elements the teacher must present in the situations which he offers pupils so as to ensure that they participate in the process and can understand sufficient aspects of the new situations proposed to make sense of them and recognize questions for themselves.

The progression of the pupils' knowledge is largely the result of their own action and personal experience and reflection; but without some help from outside, and especially from the teacher, this progression would not be so easy to achieve. The emergence of new types of knowledge and the adoption of more

appropriate types of knowledge proposed by others can be held back by epistemological obstacles. The idea of epistemological obstacle, as developed by Bachelard and taken up by the didactics researchers, is based on the thesis that knowledge that has been formed and tested in action and consolidated by experience makes the assimilation of certain concepts even more complex than a complete absence of knowledge. For example, the use of numbers to measure discrete quantities and continuous magnitudes, which are intrinsically positive, makes it more difficult to grasp the idea of negative number. Similarly, normal experience of movement supports the idea that force and speed have the same direction and makes it more, not less, difficult to see instant speed and force as independent magnitudes. The concept of obstacle is all the more important because pupils develop misconceptions spontaneously and those misconceptions often reflect certain aspects of the most familiar, and most easily controlled, situations.

This view of learning and development processes is fundamentally compatible with Piaget's theories. But Piaget stopped short of examining the idea of epistemological obstacle: his work contains no examples of such problems as those of negative number and force to which we have just referred. Nor did he develop the idea of didactic situation, despite the fact that his analysis of cognitive disequilibrium and re-equilibration processes moves along similar lines. It may be said that the subjects, at any given moment in their development, have a repertory of skills which enable them to deal successfully with a large number of situations and that they have therefore achieved a certain equilibrium between the complexity of their cognitive resources and that of the situations with which they are faced. When confronted with new situations which they are not yet able to control, they need to develop new resources, which will be the result on the one hand of adaptation through accommodation of the resources already acquired, and on the other of the discovery of entirely new properties of reality.

What are being developed? Specific conceptualizations or general structures of thinking?

Piaget studied the development of many concepts: that of number, space, time, speed, chance, proportionality, etc., and yet the aim of his theoretical work was to contribute not so much to a psychology or epistemology of the concept as to a psychology of the general structures of thinking. He felt able to identify a number of general stages, of which the last two, in his view, were characterized by structures directly inspired by logic:

— the stage of 'concrete' operations, by the concept of grouping;
— the stage of 'formal' operations, by the INRC group.[2]

This is an approach which the didacticians have not adopted, being more concerned with detailed analysis of pupils' progress through a large number of situations in which the same concept may display very different properties. Proportionality, for example, covers a vast series of problems, of which the sim-

plest are easily understood by most nine-year-olds, whereas the most complex cause problems for the majority of adults.

Here are some of the conceptual phases which have to be covered:

1. The most elementary property of the linear function:

$$f(n) = nf(1)$$

The cost of n objects is equal to n times the cost of one object. Consumption or production for n days is equal to n times consumption or production for one day, etc.

2. The most general isomorphic properties of the linear function:

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

The cost of the sum of two quantities of the same merchandise is equal to the sum of the two costs.

$$f(\lambda x) = \lambda f(x)$$

The cost of λ times a quantity x of merchandise is equal to λ times the cost of that quantity.

3. The general theorem of linear combinations which puts together both properties:

$$f(\lambda_1 x_1 + \lambda_2 x_2) = \lambda_1 f(x_1) + \lambda_2 f(x_2)$$

4. The properties linked with the constant coefficient, which are conceptually slightly more complex for primary school students:

$$f(n) = an$$

The cost of n objects is equal to n multiplied by a constant coefficient or the reciprocal equality:

$$n = (1/a)f(n)$$

One does not need to be a great intellectual to see that there is a difference in level between the procedures which are required to deal with multiplication problems involving small whole numbers, when the quantities or magnitudes are familiar, and those which apply to the search for a fourth proportional, when numbers are very large or less than 1, and the quantities or magnitudes are unfamiliar. These differences of level reflect an ability to deal with concepts which varies in depth, generalization and flexibility: the ability to match the procedure to the situation indicates a greater ability to control that situation. But the ability to conceptualize proportionality does not stop there, since pupils very soon come into contact with ways of combining proportions which, although less complicated than those encountered in physics, with equations involving dimensional analysis, nevertheless cause much greater difficulties than the situations involving proportions between the two variables referred to above.

There are in fact two ways of combining proportions: by concatenation and by product.

5. The concatenation case may be expressed by a composition of linear functions. For example, in concrete manufacturing, the quantities and prices are all proportional one to the other: concrete in m^3/cement in kg/sand in kg/gravel in kg/cost of cement/cost of sand/cost of gravel/cost of concrete.

Whatever the two linked linear functions f and g may be, the composition $f o$

g is linear. One can always, therefore, have recourse to the properties of the above-mentioned linear function.

6. The product of two proportions makes it necessary for students to understand the more subtle concepts of dependence and independence. A variable Z is proportional to a variable X when a third number Y is constant; and it is proportional to Y when X is constant. For example, a community's consumption of bread is proportional to the number of people as long as the duration is constant, and to the duration as long as the number of people is constant.

The relevant mathematical model is not the linear function but the bilinear function. Some of its properties are similar to those we have already seen:

$$f(\lambda_1 x_1, x_2) = \lambda_1 f(x_1, x_2)$$

λ_1 times more people ⇨ λ_1 times more bread;

$$f(x_1, \lambda_2 x_2) = \lambda_2 f(x_1, x_2)$$

λ_2 times more days ⇨ λ_2 times more bread.

But a new property appears:

$$f(\lambda_1 x_1, \lambda_2 x_2) = \lambda_1 \lambda_2 f(x_1, x_2)$$

which transforms the double proportionality into a structure which is very useful for spatial measurements (area, volume) and physical measurements (mechanics, electricity).

The example of proportionality, despite the brevity of the explanation given here, demonstrates the futility of reducing such a wide range of conceptual steps to one particular stage in the development of thought processes, still less to one particular logical structure, since it extends over a long period of learning and development which begins at the age of 8 and covers many years. It is more useful to analyse the procedures applied by students to an organized variety of situations, so as to try to see when filiations and ruptures take place in the process of conceptualizing multiplicative structures and proportionality.

This process stretches over a period of several years and is very dependent on the guidance provided by teaching. The obstacles encountered by pupils have now been identified, for example those involved in multiplication and division by a number less than one. Errors in reasoning are themselves more easily described when linked to the relevant mathematical concepts, namely linearity and dimensional analysis. The INRC group contributes almost nothing to the analysis.

My conclusion on this point is therefore clear. What creates the unity of a process of conceptualization is more the longitudinal links than the transversal association. The conceptual area of multiplicative structures and proportionality, which is mastered by students and adults in a slow and gradual process, provides a more operational theoretical framework for research and teaching than the logical structure which is supposed to characterize the 'formal' operatory stage.

How to analyse action in a given situation and the conceptualization underlying action?

This is probably the question in most need of an answer today, as most of our knowledge consists of competences and these competences form, develop, are differentiated and improved, and possibly deteriorate during our life depending on the range of situations to which we are exposed. With the concept of *scheme,* Piaget offered the most decisive concept on this point, even if he did not emphasize it as much as would seem appropriate today. Three basic ideas, already present in Piaget, can be taken as starting-points:

- that a scheme is a functional dynamic whole;
- that good examples of schemes can be found in the area of so-called 'sensory-motor' activity;
- that schemes are relevant not only to sensory-motor but also to intellectual activity: this is the case in particular for classification activities and logical reasoning, and Piaget explicitly mentions the scheme of proportionality (which must obviously be put in the plural today).

The schemes observed by Piaget in babies concern certain instinctive behaviour patterns such as suckling and sucking or opening and closing the hand, and some more elaborate ones such as shaking a rattle to make a noise, transferring an object from one hand to the other, and pulling a piece of material to bring something placed on it nearer. To these may be added many other gestures and movements of the body. For each of the examples, a degree of organization of the activity may be observed, which is related to its function and its temporal progress. This totality is of course analysable but the different elements only become meaningful in relation to the organization and function of the whole.

Today it is possible to go considerably further than Piaget in seeking examples and in analysing and defining the concept of scheme.

We can now examine even more instructive examples such as movements which are difficult to learn—e.g. certain movements that have to be mastered in high-level sport or ballet dancing and in certain jobs. We must also look at activities usually regarded as intellectual rather than gestural—e.g. counting in young children, tracing figures with a set square and compass, writing—or even activities wrongly considered as purely intellectual, such as solving mathematical problems or knowing how to tell a story, how to join in a conversation, how to manage situations of co-operation or conflict with others. These are skills organized by schemes and learned largely through experience, if not by a deliberate learning process.

For the purposes of analysis, they must be related to the characteristic features of the situations to which they apply. A scheme may be defined *as an invariant organization of behaviour for a given set of situations.* This organization is based on four main types of element:

- goals and expectations;

— rules for action, information taking and control;
— operatory invariants; and
— possibilities of inference.

Their specific functions may be analysed as follows:

A scheme always applies to a type of situation in which the subject can identify a possible target for his/her activity, and sometimes intermediary sub-targets too. In addition, certain side effects or phenomena may be anticipated.

The rules of action are the generative part of the scheme, engendering a series of activities aimed at transforming reality, seeking information and controlling the outcome of the activities; this makes it possible to ensure the success of the activity in a context that may be constantly evolving.

The operatory invariants form the implicit, or explicit, conceptual basis which makes it possible to select the appropriate information and, by inference based on that information and the target to be attained, to identify the most appropriate rules of action. We shall return later to the two main categories of operatory invariants: concepts-in-action and theorems-in-action.

Lastly, schemes inevitably include possibilities of inference, since all the above-mentioned activity requires calculations, carried out *in situ*. A scheme is not generally a stereotype, but on the contrary an instrument for adapting activity and behaviour to the specific values acquired by different parameters as the situation develops.

Defined and analysed in this way, schemes are appropriate for all the possible registers of behaviour, including competences as different as physical gestures, intellectual, scientific and technical activities, interaction with others, affectivity and the use of language. They may also apply to different levels of organization: the use of language, for example, will involve schemes for phonology, vocabulary, syntax, tone and discourse organization in a dialogue.

It may be regretted in passing that Piaget, like his contemporaries, used the expression 'sensory-motor' to designate a sphere of activity which is in fact 'perceptory-gestural' since it is organized by underlying percepts, gestures and schemes.

The main attraction of the scheme concept for theoreticians is that it provides the essential link between behaviour and representation. It is in fact the operatory invariants which form the main linkage, since perception and information seeking and selection are based entirely on the system of the concepts-in-action at the disposal of the subject (objects, attributes, relations, conditions, circumstances, etc.) and the theorems-in-action underlying his or her behaviour.

A theorem-in-action is a proposition concerning reality which is held to be true; and a concept-in-action is a category of thought that is held to be relevant.

For instance, the above example of proportionality, $f(\lambda a) = \lambda f(a)$, is a theorum-in-action, and the scalar λ is a concept-in-action.

Significance for education and training

If most of our knowledge consists of competences and is thus made available in the form of schemes, education should attach greater importance than it does officially to their formation and to the situations which make them functional. The development of active teaching methods and research by didacticians have helped to make this more widely recognized. There are still some grey areas, however, since the role of conceptualization is not always given its proper place, and teachers' actual behaviour in the classroom bears little resemblance to their declared intentions. Piaget probably offers the best theoretical basis for this realization, even if he did not go as far as he might have done.

For example, Piaget attaches very little importance to the concept of *situation;* yet given the importance he attaches to the concept of scheme one wonders why he did not establish a close correspondence between these two concepts. In his *operatory theory of representation:* Piaget speaks of subject-object interaction, when he could have been more precise and spoken of the *scheme-situation interaction.* A theory of representation needs a theory of reference, and the reference, in the subject's representation of the world, does not consist only of objects and their properties but also of situations in which his or her activity is involved, and which provide him/her with the basis for organization of his/her activity.

In short, the idea that cognitive development consists first and foremost of the development of a vast array of schemes has now become acceptable. They cover very different spheres of human activity. An individual's professional competence often consists, in addition to purely technical and scientific skills, of skills in managing affects and social relations which carry considerable weight. Education and training should therefore help to create a diversified array of schemes, taking care that they do not petrify into stereotypes.

Repetition is an important aspect of the formation of schemes, since it is the familiarity of the situations which make the most decisive contribution to the process. But repetition can be dangerous unless there is a degree of variation. A scheme is a universal which can be applied to a class of situations. Its flexibility depends on the variety of the situations contributing to its formation. For example, addition and subtraction involve a very wide variety of classes of problems, and pupils are not sufficiently prepared to deal with them if they are only given a restricted range of cases. They need also to be given the means of rejecting the narrow and erroneous conceptions which form spontaneously during their first contact with addition and subtraction and which they therefore have difficulty in rejecting: 'addition is an increase, and subtraction is a decrease'. This conception is wrong and makes it difficult to solve those addition and subtraction problems which contradict it.

Similarly, the operations involved in understanding a text vary from one type of text to another. Different schemes are involved in understanding a story, a theatre or film dialogue, the statement of a mathematical problem or a chapter in

a geography book. They have different goals, different rules for assimilation and control, and different operatory invariants.

Is this enough?

Obviously not! Analysis of the skills underlying behaviour is one thing; it does not exhaust either the characteristics of knowledge presented in textbooks or the psychological phenomena which take place in the classroom, notably reflexive acts of awareness and acts of mediation by the teacher. Piaget had several important ideas to contribute on these two points, but we clearly need help from Vygotsky and Bruner to complete the picture.

It is in fact Vygotsky who states most clearly that scientific concepts:

— have a general validity far greater than that of the concepts applied in everyday life; they also form integrated systems, whereas everyday concepts only have local impact;
— are transmitted to pupils in school, as one of the school's main functions, through the deliberate action of a teacher, whereas everyday concepts are formed spontaneously by the child on the basis of his/her experience;
— are communicated mainly by the use of language and other symbolic means such as those of graphic representation, whereas everyday concepts remain largely implicit and can even be 'unconscious'.

Perhaps Vygotsky makes too much of this opposition, and his examples are not very well chosen (kinship relations taken as an example of everyday concepts cannot be transmitted without the use of language, and they form an integrated system), but it must be recognized that the distinction between everyday concepts and scientific concepts makes a very useful contribution to analysis of teaching and learning in the classroom. The cognitive status of the operatory invariants, the rules for action, the goals, anticipations and inferences change as soon as they are expressed. An explicit proposition can be discussed, but a proposition accepted as true in a totally implicit way cannot. The nature of knowledge therefore changes if it is communicable, discussed and shared.

Piaget showed little interest in forms of conceptualization involving the use of language. And yet they make it possible to put forward important ideas which complement those contained in his operatory theory. In other words, understanding of teaching and learning problems is based on analysis of both predicative and operatory forms of knowledge. For example, the conceptual level of a given concept changes depending on whether the concept is incorporated in a statement in the form of a noun (when it is the object of thought and the subject of the assertion), or in the form of an adjective, a verb or a relative word (when it is the predicate). This Knowing how to calculate speed by dividing distance by time is not the same level of conceptualization as expressing through language the idea that speed is proportional to distance when time is constant and inversely proportional to time when distance is constant, or else saying that distance is a bilinear function of time and speed

and expressing the latter either by a formula $D = ST$, or by a double proportion table.

Similarly, it is no longer possible to pass over the idea of mediation, with the double meaning which Vygotsky attributed to it: the teacher is a mediator, language is also a mediator. Educational research, and in particular didactical research, has not paid sufficient attention to the analysis of mediation phenomena, despite Bruner's magisterial presentation of them in his analysis of acts of mediation between mothers and their babies. Vygotsky may have minimized the role of the subject's own action during the learning process, but Piaget seems to have minimized the role of the mediation provided by adults and other children, as well as the role of language.

This being said, an attentive reader is more often struck by convergences than by divergences between Piaget and Vygotsky—the doyens of the psychology of the higher cognitive activities. For example, both devoted considerable attention to the action of becoming aware and to reflexive awareness: in short, it may be said that the Vygotskian distinction between 'awareness before' and 'awareness after' is similar to Piaget's distinction between 'succeeding' and 'understanding' or between 'simple abstraction' and 'reflective abstraction'. Similarly when Vygotsky, referring to children's use of language and opposing Piaget's idea of egocentric language, supports the view that the child *internalizes* dialogue with others, he is, paradoxically, concurring with another well-known theory of Piaget, clearly set out in his work *La formation du symbole,* namely that representation is the *interiorization* of action and perception.

Lastly, the years of work devoted by Piaget and his school to the methodology of clinical and critical interview are clearly a contribution to the analysis of mediation.

What is the heritage for didactics?

Neither Piaget nor indeed Vygotsky or Bruner really laid an adequate basis for the new fields of research known as didactics which have developed over the last twenty years. We shall now seek to identify some of the elements which made it possible to move from the penetrating insights of Piaget and Vygotsky to the analysis of the teaching and learning processes as they are presented today in didactical research. Didactics is a field of research which is still in its infancy and our concluding remarks must therefore be taken with caution.

Firstly, explicit reference to established disciplines and their epistemology is crucial for didacticians, less on the basis of psychology than on that of other approaches, however. For instance, in the didactics of science and mathematics teaching, works of history and epistemology such as those of Bachelard, Koyré or Canguilhem in France are likely to be referred to. Didacticians are interested in the learning and teaching of specific areas of knowledge; they therefore need specific references either to disciplines and their history and to professions and the techniques to which they have recourse.

Secondly, reflection on actual teaching practice, based on the difficulties teachers themselves encounter, has provided a number of important ideas from which a number of didactic scenarios have been derived, as well as analysis of errors and of certain phenomena which break the didactic contract. Research has profoundly changed the way in which teachers think, but their thinking remains a rich source of ideas when considered in the light of sociology, history, epistemology and psychology.

However, the most decisive step in the emergence of didactics as scientific disciplines was experimentation work in class, with carefully thought-out and prepared situations, specially trained teachers, and methods of observation, recording and analysis which neither Piaget nor Vygotsky had thought to set up. Didacticians themselves had to get rid of some preconceptions which hindered rather than helped the analysis of pupils' skills and the circumstances in which those skills were acquired. We need not refer here to behaviourism or intelligence tests, which held development in educational research back for many years. Piaget's exposition of the general stages of thinking seem to me to have been more of a hindrance than a help in the development of didactics, insofar as it defines, as general logical operations, conceptualizations which cannot be reduced to logic.

The heritage of a pioneering spirit such as Piaget deserves examination that is both attentive and critical. The originality and importance of his works earned him both tremendous support and keen criticism. The time has come to rise above both these attitudes and to consider in an objective way first of all what constituted a decisive contribution, to be assimilated by all, and what, on the other hand, seems on reflection to be less important and no longer so relevant.

Piaget provided several cornerstones for didactical research: the ideas of adaptation, disequilibrium and re-equilibration, a great many points concerning the conservation of quantities, spatial representation and the formation of mathematical and physical concepts, etc.

However, the keystone is probably the concept of scheme. I have tried in this article to propose an analysis of this concept which would make it possible to apply it to a wide variety of behaviour patterns and activities. It is in fact a key to analysis of both activities involved in representation that are directly observable and those that are not directly observable. It is a concept which has much to contribute to all levels of human activity and offers by far the most helpful psychological tool for the study of adaptation.

Notes

1. By 'operatory thesis' we mean simply that knowledge is formed, developed and transformed by means of action; the term 'operatory', as used by us, does not therefore convey all the ideas which Piaget associated with it: a distinction between action and operation, a structured and reversible set of operations, etc.
2. INRC group: Piaget used to consider that the formal stage of cognitive development

could be described as the co-ordination of the relationship between four transformations of sentences: N = negation; R = reciprocation; C = correlation; I = identity. They form a Klein group.

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TRENDS/CASES

GRADE RETENTION PRACTICES IN PUBLIC AND PRIVATE SCHOOLS IN LEBANON

Karma A. El-Hassan

Grade retention is the practice of requiring a student to repeat a year of academic instruction at a particular level (Jackson, 1975). There has been a recent increase in retentions in the United States (Shepard & Smith, 1990) and in many developing countries (Amadio, 1995). Holmes (1989) notes that an estimate of a 6% annual retention rate for public schoolchildren produces a cumulative rate of non-promotion greater than 50%. Between 1970 and 1980 repetitions increased by 41% in Africa, 46% in Latin America, and 7.3% in Asia (Amadio, 1995). This increase is related to the back-to-basics movement and the trend towards minimum competencies (Niklason, 1984).

Educators who favour the use of grade retention claim that it serves two major purposes: to remedy inadequate academic progress; and to enhance the development of emotionally immature students (Jackson, 1975). They maintain that re-exposure to academic material and placement in a group of younger children are beneficial educational interventions for the failing child. This practice is recommended despite the fact that accumulated research fails to provide evidence that retention can be beneficial, rather suggesting that it can be harmful (Holmes & Mathews, 1984; Jackson, 1975; Johnson, Merrel & Stover, 1990; May & Welch, 1984; Meisels & Leaw, 1993; Morris, 1993; Niklason, 1987; Rose et al., 1983). The general conclusions of these reviews is that promoted students perform better than children who are retained on measures of academic achievement, personal adjustment, self-concept and attitudes towards schools (Roderick,

Karma A. El-Hassan (Lebanon)
Assistant professor in the Division of Education Programs and Director of the Office of Tests and Measurements at the American University of Beirut. Ph.D. in educational psychology from the Institute of Education, University of London. Research interest in faculty evaluation, large-scale assessment and pre-school assessment. A member of several international testing and research organizations. Has published on status of testing in Lebanon, on university admission tests and on faculty evaluation.

1994). Retention also increases the risk of dropping out (Grissom & Shepard, 1989) and the financial and psychological costs of the problem thus become very significant, as retention requires more years to complete the same amount of learning and this depletes the scarce resources available for education.

With respect to Lebanon, retention has always been and still is a major problem rooted in an outdated education system (Jibai & Abu Rjaili, 1993). In addition to this is the fact that schools, at the end of the sixteen-year civil war, started grade retention in 1989 as a means of regaining standards lost during the war. No recent research has yet been done in Lebanon to investigate the extent of the problem. The last serious effort in this respect was carried out in the 1970s. UNESCO and the Centre for Educational Research and Development (CERD) conducted two studies at that time that reported high retention levels, especially at the elementary level, ranging between 21 and 24% (UNESCO, 1980, as cited in Jibai & Abu-Rjaili, 1993; Antun & Abu-Rjaili, 1975). Since then, only unofficial reports and studies involving selected schools or groups of schools have been reported revealing high retention levels, especially at the end of the intermediate years. In 1992, 45.6% of students in the fourth intermediate class failed the official examinations and had to repeat their class (Lebanon. Ministry of Education, 1992).

Therefore, and given the lack of adequately designed studies and the large number of students affected, an investigation of the retention practices in schools in Lebanon is highly relevant to current educational study. In addition, and as record keeping on retention is poor (Shepard & Smith, 1989; Natale, 1991) and structural patterns of retention rates across grades and levels have not been examined (Morris, 1993), this study aims at identifying differences in grade retention (if any) by grade, educational level (elementary, intermediate, secondary), sector (public vs private), sex and geographic region.

Method

SUBJECTS

Enrolment, promotion and retention data were collected on 83,989 students, 44,023 males and 39,966 females (11% of the school population) enrolled in 265 schools and who were proportionally representative of the 1993/94 school population in Lebanon. The sample was stratified by geographic region (five districts), school type (public, private, semi-private) and school size. When the number of schools needed in every strata was determined, schools to be surveyed were randomly selected. Table 1 describes the final sample on which data were collected.

PROCEDURE

Data were collected by a group of trained field investigators during the period January–April 1995. To aid them in their work, the investigators were given

TABLE 1: Breakdown of sample with respect to school type, geographic region and sex

	Public				Private				Semi-private				Total		
	Male	Female	Total		Male	Female	Total		Male	Female	Total		Male	Female	Total
Beirut (15%)	761	1314	2075		6031	2381	8412		1019	1295	2314		7811	4990	12801
Mount Lebanon (32%)	2758	2796	5554		8687	10671	19358		1005	801	1806		12450	14268	26718
South (16%)	2768	2959	5727		3333	2294	5627		1069	835	1904		7170	6088	13258
North (24%)	3838	3880	7718		5592	4147	9739		1647	1433	3080		11077	9460	20537
Bekaa (13%)	3067	2222	5289		2573	2215	4788		875	724	1599		5515	5160	10675
Total (100%)	12192	13170	25362	(30%)	26216	21708	47924	(57%)	5615	5088	10703	(13%)	44023	39966	83,989

two letters: one from the Director General of the Ministry of Education requesting the schools' co-operation; and the other from the Principal Investigator to the principals of these schools explaining the purpose of the research and soliciting their help. Each investigator met with the school director and collected information from the school records. Some schools did not want to participate and other schools with similar characteristics were chosen at random to replace them. Some schools, mostly from the private and semi-private sectors, did not allow the field investigators to look into their records and thus filled out the information themselves, signing a form stating that they assumed responsibility for the accuracy of the data. The data were collected in the form of a 'school data questionnaire', which included information on type of school, geographic location, enrolment, promotion and retention figures for 1993/94 by grade and by sex.

DATA ANALYSIS

Retention levels, determined by computing the proportion of retained students to the total student population, were reported for the whole sample and by sex, grade, educational level, school type and geographic location. In addition, group differences in retention levels were investigated using chi-square analysis.

Results

RETENTION LEVELS

Of the investigated sample of 83,989 students studying in Lebanese schools in 1993/94, 15,806 were retained in their classes, i.e. a 19% retention level for grades kindergarten I to secondary III. If kindergarten is excluded, the retention level rises to 21%. Males had a higher retention rate than females for the whole sample (20% versus 18%). Retention levels by geographic region, school type, educational level and grade are reported in Tables 2 to 5 respectively.

TABLE 2: Percentages of retention in each geographic region by gender and school type

		Sex		School type		
Region	Total[1]	Male	Female	Public	Private	Semi-private
Beirut	17	16	18	35	14	10
Mount Lebanon	16	18	13	34	10	15
South	22	22	21	34	11	14
North	21	21	20	40	8	12
Bekaa	23	24	22	36	15	14
Total	19	20	18	36	11	13

1. ($X^2 = 450.82$; $p<.0001$)

TABLE 3: Percentages of retention in each school type by gender and geographic region

	Sex			Region				
Type	Total[1]	Male	Female	Beirut	Mount Lebanon	South	North	Bekaa
Public	36	39	33	35	34	34	40	36
Private	11	12	10	14	10	11	8	15
Semi-Private	13	14	12	10	15	14	12	14
Total	19	20	18	17	16	22	21	23

1. (X^2 = 7040.86; p<.0001)

TABLE 4: Percentages of retention by educational level

	Sex			Type			Region				
Level	Total %	Male	Female	Public	Private	Semi private-	Beirut	Mount Lebanon	South	North	Bekaa
Kindergarten	4	4	5	11	3	4	1	5	4	55	5
Elementary	17	19	15	35	9	15	12	14	18	21	22
Intermediate	27	28	26	42	17	14	27	22	32	29	31
Secondary	24	30	19	37	16		29	19	35	15	36
Total	19	20	18	36	11	13	17	16	22	21	23

TABLE 5: Percentages of retention by grade

	Sex			Region					Type		
	Total[1]	Male	Female	Beirut	Mount Lebanon	South	North	Bekaa	Public	Private	Semi Private
KGI	3	2	8	3	4	3	2	4	5	3	4
KGII	5	5	4	2	5	4	6	6	12	3	3
Elem. I	11	13	9	7	8	11	15	15	25	5	13
Elem. II	16	19	14	10	11	18	22	22	35	7	16
Elem. III	19	21	16	12	15	21	22	23	37	9	16
Elem. IV	21	23	19	18	17	20	25	26	38	13	17
Elem. V	20	21	19	11	20	22	23	22	36	13	13
Int. I	28	30	25	27	23	32	32	30	44	17	17
Int. II	22	23	22	24	19	27	21	26	36	14	18
Int. III	23	24	22	22	18	29	23	28	35	17	
Int. IV	36	36	36	39	28	42	41	43	55	22	7
Sec. I	22	28	16	25	18	35	10	30	34	14	
Sec. II	18	21	16	18	15	28	3	32	28	11	
Sec. III	35	43	27	43	24	51	36	48	51	25	

1. (X2 = 3599.59; p< .0001)

GROUP DIFFERENCES

Comparison between retained and promoted students with respect to sex, school type, geographic location and grade were performed using chi-square analysis. There were significant sex differences ($X^2 = 66.19$; $p<.0001$) between promoted and retained with a higher proportion of boys kept back than girls. There were also significant differences in the proportions of children retained and promoted by school type ($X^2 = 7040.86$; $p<.0001$) with the public schools showing highest retention levels (36%). Similarly, chi-square analysis revealed significant differences between geographic regions ($X^2 = 450.82$; $p<.0001$), with the more rural areas (North, South, Bekaa) showing significantly higher retention rates (21–23%) than the more urban areas (Beirut, Mount Lebanon, 16–17%). Grade retention levels ranged between 3% in kindergarten 1 to 36 % at the end of the intermediate level (Table 5) and chi-square analysis revealed the differences to be significant ($X^2 = 3599.59$; $p<.0001$). These grade retention levels increased gradually with a pronounced tendency to show a large increase at the beginning of a major organizational change, i.e. levels of schooling at the beginning of elementary and intermediate. For grade 1, retention rose to 11% compared to 3% in kindergarten 2. Similarly, for the first intermediate level, retention rose to 28% from 20% in the preceding fifth elementary, then it started to decline until it peaked again at the fourth intermediate (36%) because of the official government exams at the end of this grade. Education level retention rates also showed a gradual increase from 4% in kindergarten to 27% for the intermediate level. The drop in secondary level retention to 24% is probably due to the fact that many of the retainees drop out at the end of the intermediate level.

Discussion

This study surveyed retention practices in public and private schools in Lebanon for 1993/94 by collecting data on 83,989 students representative of the school population in Lebanon. Average annual retention rates for kindergarten grades through to the third secondary were estimated at 19%. Retention levels were also reported by gender (males 20%; females 18%), by school type (public 36%; private 11%; semi-private 13%), by geographic region (rural 21–23%; urban 16–17%), educational level (kindergarten 4%; elementary 17%; intermediate 27%; secondary 24%), and by grade level (3–36%). The results of chi-square analysis showed that retention decisions were not independent of gender, school type, geographic location and grade-level placements.

The estimated 19% annual retention rate for kindergarten to secondary grades is very high compared to the United States and to regional standards. Rose et al. (1983), in an overview of prevalent retention rates for 1978–80 for several states, found that those for combined grade levels 1–12 ranged from 4 to 8%. However, according to Shepard & Smith (1989), the number of retentions has increased substantially to 7–9% in schools, especially in large cities, where achie-

vement-based promotion policies have been introduced. Similarly, for the Arab countries, the retention levels range between 3 and 9% for primary and 6 and 14% for secondary (United Nations Development Programme, 1994; UNESCO, 1993). The only exception is Iraq, with retention levels of 19% and 32% at the primary and secondary levels respectively. As far as Lebanon is concerned, the UNESCO and CERD studies, carried out on samples of children in 1968/69 and 1972/73 respectively, showed average retention rates of 16.5% (UNESCO as cited in Jibai & Abu Rjaili, 1993) and 15% (Antun & Abu Rjaili, 1975). These two studies covered grades I–IV at the intermediate level. Likewise, if we exclude the kindergarten and secondary grades from our figures, the retention rate for our sample of intermediate grades I–IV will become 22%. Although different methodologies were used in these studies, especially in the CERD study, we can still safely conclude that retention levels in Lebanese schools have risen and are very high compared to regional and international standards. The best possible explanation could lie in the fact that during the war years—1975–89—educational standards dropped and social promotion was the common practice (Jibai & Abu Rjaili, 1993). Schools therefore are now resorting more and more to the practice of retention to raise standards.

With respect to retention level by grade and with advancing years, rates increased gradually with significantly high jumps at the beginnings of elementary and the beginning of the intermediate years (Table 5). This is slightly different from the pattern found in the United States, which showed that rates start very high in the first grade then drop rapidly to the very lowest rank by the fifth or sixth grade and peak again at the seventh, i.e. the beginning of middle school (Rose et al., 1983). Morris (1993) suggests that these retention rate peaks might reflect difficulties in coping with changes in school environments from one level to the next. This poses a problem of adjustment to a changing school size which lowers self-esteem and subsequently affects school performance. Walker (1984) confirms the high retention rate of first-graders and quotes an average retention of 12% to 18% based on the 1971/72 survey, while Meisels & Leaw (1993) quote a range of 5 to 14% retention level of first-graders based on a 1987/88 sample. In Lebanon, both the CERD and UNESCO studies noted high retention rates (21–23%) in grades 1 to 3 but these rates start to decline with advancing years. This common retention of first graders is an attempt by school systems to prevent school failure by retaining a large number of first graders who are deficient in basic skills. The gradual increase in retention rates in this study from intermediate grades I to IV can be explained by two facts. First, with the resumption of the official (government) examinations, especially at the end of the intermediate years, schools have resorted to stricter promotion requirements to ensure success. Second, this movement was accompanied by a lower drop-out rate, especially at the elementary level (Jibai & Abu Rjaili, 1993), so the at-risk students stayed in the system and were more likely to be retained again, thus raising the retention level with advancing years. Straits (1987) cites several studies which show 'that age grade level retention is a cumulative or snowballing process'

(p. 40) implying that the same retained students may soon again be candidates for retention.

The higher incidence of male grade retention found in this study is supported by research findings (Abidin, Golladay & Howerton; 1971; Antun & Abu Rjaili, 1975; Jibai & Abu Rjaili, 1993; Mantzicopoulos et al., 1989; Meisels et al., 1993; Niklason, 1984). A possible explanation could be that males have a higher incidence of attention problems than girls (Mantzicopoulos et al., 1989). Similarly, differences in retention rates between public and private schools and between urban and rural areas found in this study are well documented by research findings. The CERD study confirms the sex differences in retention as well as the differences between public and private schools. However, the difference in retention levels between urban and rural areas is less significantly evident (Antun & Abu Rjaili, 1975). Nevertheless, students studying in public schools and in rural areas tend to come from a lower socio-economic level than those in urban and private schools, and low socio-economic status was found to be associated with high retention rates (Mantzicopoulos et al., 1989; Meisels and Leaw, 1993; Niklason, 1984). The UNESCO report on school repetition confirms this by concluding that 'all the studies carried out in different regions agree that the highest repetition and, more generally, school failure rates are observed among students from poor families, from marginal social groups with low standards of literacy, in rural environments or in areas which are socio-economically and educationally backward' (Amadio, 1995, p. 5). Meisels and Leaw (1993) estimate that four times as many of the retainees come from the lowest socio-economic group as compared with the highest (34% versus 9%) and that they are more likely to be boys (24% versus 15%). Furthermore, research has extensively reported that boys from low socio-economic origins are often regarded as less able by their teachers, regardless of their actual abilities (Alexander & Entwisle, 1988). The higher incidence of retention associated with children from low socio-economic groups is due to a poor environment which does not favour the normal physical and mental development of the child and which is also lacking in teaching facilities and resources, parental education, interaction and expectations—the proven correlates of poor school performance.

With the paucity of information on retention practices in Lebanon, this study has attempted to give a recent and extensive idea of the quantitative dimension of the problem and some of its qualitative characteristics. It conforms to the conclusion of the UNICEF/IBE meeting on grade repetition, held in Geneva in February 1995, that 'suitable strategies have to be based on sound diagnosis' (Amadio, 1995, p. 7). The estimated high retention level present in Lebanese schools should alert officials and school administrators to the resulting economic wastage, as well as to the negative educational and psychological effects of retention. In addition, special attention should be directed to the public sector and those geographic regions where higher retention rates prevail.

One limitation of this study is that it gives an estimate of retention during one year only, namely 1993/94. Future research should be directed towards stu-

dying the levels over several years in order to come up with a more stable and accurate estimate. Another limitation is that some of the schools, especially in the semi-private sector, filled out the information sheets themselves, pledging responsibility for the accuracy of their information. Yet it can be argued that they may not have supplied reliable retention figures. Future research should investigate other approaches to the measurement of grade retention. More importantly, it should be directed towards identifying the factors—demographic and educational—that contribute to a high retention level in Lebanon. It should also aim at assessing teachers', principals' and parents' beliefs about retention, as these are a decisive factor in retention decisions. Finally, it should explore and recommend research-based alternatives to retention that are appropriate to the Lebanese situation. Detailed strategies and plans have to be devised to deal with the problem of grade retention in Lebanon. This study hopefully lays the grounds for such action.

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PROFILES OF FAMOUS EDUCATORS

AUGUSTE COMTE

(1798–1857)

Jacques Muglioni

A strange destiny! The whole of Comte's life is a romance. His successors are divided between two attitudes—the incomprehension of disciples of limited outlook, and the indifference, or even hostility, of thinkers who could have learned a great deal, if only they had read him.

The mathematician in him fulminated against the limitations of calculus: he was as severe with the scientists of his time as he was with the writers and journalists. As founder of positivism, he went so far as to conclude that science was a thing of the past, whereas the future belonged to art.[1] He saw in the French Revolution the greatest event in universal history, while at the same time he thought only of putting an end to the crisis it had brought about. This examiner at the Ecole Polytechnique extolled pure love. This man of faith announced the end of superstition and, at the same time, instituted the Religion of Humanity. He condemned the abstract feminism of the 'negative' period, but only in order to give women a leading place in a regenerate humanity. His admirable correspondence[2] with Clotilde de Vaux during the 'year without parallel' (1845) was to prompt Alain to say that Musset's *Les nuits* were indeed read!

The difficulty is how to read the ten volumes of Comte's work without bothering about the commentaries, most of which are mediocre or even mendacious. The reader is rewarded at times on discovering a lucidity, on the verge of madness, found only in Kant and Hegel, who, apart from Comte, are perhaps the last great names in the history of philosophy as inaugurated by Plato. Thus, we are in the presence of an author who will be a closed book to readers who are devotees of modernity.

Jacques Muglioni (France)
Agrégé de philosophie, taught philosophy in the final year of the secondary level in Paris and elsewhere in France, then at the upper preparatory level at the Lycée Henri IV, Paris, before assuming the duties of dean of the general inspectorate of philosophy. Author of a number of articles, some of which were recently collected in two volumes: *L'école ou le loisir de pensée* (1993) and *Auguste Comte, un philosophe pour notre temps* (1995).

Ethics and polity

The decisive point, no doubt, is that Comte's work leads not so much to a political doctrine as to a philosophy of education. As early as 1825 he was to write, 'Education and philosophy are closely and necessarily related'.[3] Actually, the dominant question in his work arose from the fact that the old spiritual power which regulated pre-Revolutionary society in essential matters had collapsed. As Catholicism had become no more than an imposing historical ruin, the moral ascendancy it exercised in the Middle Ages had become the task of journalists and literary writers, the spiritual guides of modern times. Nowadays we would say that for the general public the media had replaced the Church. Humanity thus appeared to be lost unless some new ascendancy, of positive inspiration, emerged to regulate contemporary society and so to ensure the future of humanity. Hence, education was not a function like any other: it was the very essence of society. For the most part, people behaved as they had been brought up to behave.

This idea, which inspires all of Comte's thinking, suffered a long eclipse, which may not even now be nearing its end. Montesquieu and Rousseau, after Plato, were well aware that the public institution was a dead letter without the virtue of the citizen, and therefore that polity relied above all on education. However, the nineteenth century, fascinated by industrial progress and the law of supply and demand, finally preferred the empiricism of the English-speaking world, and subsequently tended to subordinate thought to economic mechanisms. In this respect, at least, Marxists and liberals agreed to share the same assumptions—a fact well illustrated today by the spurious political debate. Conversely, the decisive role accorded to education by Comte presupposes the independence of spiritual power from temporal power: we might say the independence of intellectual authority from political power. Thus Comte, at a far remove from the confusion caused by the ideas in vogue, was able to distinguish, and even contrast, authority and power.

It was not therefore the spiritual which mirrored the temporal, as if it were a mere 'superstructure', but rather the reverse. People usually behaved in accordance with what they believed, with the prejudices or convictions current in the world in which they grew up. Hence, the urgency of a new spiritual power which, contrary to the journalism then serving as a spiritual guide, could make it possible to overcome the crisis resulting from the disintegration of the old system. We know that, according to the 'Law of the Three Stages',[4] a crisis is the intermediate and transitory stage between two orders, the first of which, the *theological stage,* having lapsed, awaits a subsequent, positive stage. Now, the new spiritual power capable of inspiring education from that point on, far from being a mere reflection of existing society, should enable that society to leave behind what Comte calls *the metaphysical or abstract stage,* which in his mind stood essentially for the crisis brought about by the disintegration of the old system.

Intelligence and learning

Thus understood, education, whose role was decisive for the future of humanity, was naturally addressed to the individual. Its effectiveness could vary considerably, however, not so much because of social differences, as was to be thought later (and even more so in our own time, when people persist in subordinating virtues to situations, as Alain would put it), as by reason of the relationship with nature, which remains at the root of individuality. That is why, when Comte defined humanity, he made it clear that 'all humanity' could not mean 'all human beings',[5] but only those capable, to some extent, of partaking of the essence of humanity by virtue of their contribution, of whatever nature, to the common task. Human beings could not be reduced to animality, but their organic basis gave them an indestructible temperament, although more often than not it was perfectible. An individual was therefore far from being a simple result of the environment. Taking from phrenology what was worth retaining, as Alain suggests,[6] it had to be concluded that education might have its limits. Between those who would never really be 'persons' and the great figures of genius, there were many different degrees. Comte's thought will be seen to be far removed from current thinking, dogmatically put about by the media, concerning 'underachievement at school' and 'socio-cultural handicaps'.

Even then it had to be made clear that intellectual success was far from being the absolute criterion, or even a necessary one. Many a worker was worth more than certain doctors. Intellectual merit itself was not to be measured merely by academic success. Not only was it essential to distinguish between intelligence and learning,[7] but it had to be remembered that there was a talent that preceded education, as was clearly to be seen in the fine arts. A person's skill, humanity, and also good judgement, could not be sanctioned by certificates or degrees. There again we see how Comte would have treated the question raised by inequality of academic achievement, which spurs our reformers to reduce the school to a day-care centre without anyone in charge, or again the question of exclusion, which sanctions the lumping together of the real victims of social injustice and the parasites of humanity that 'proliferate'[8]—Comte expressly noted— 'in anarchical times'. He likewise said that there was no such thing as mathematical, or physical, or chemical genius, as if the 'partitioning of the brain-pan' were to be taken seriously;[9] on the contrary, it is the whole person that makes a choice and takes up a vocation.

History and psychology

History was the great educator of humanity. History was education, as so well shown by the theory of language[10] and that of art.[11] Humanity educated itself over time by a kind of 'self-creation', which, far from expressing an arbitrary freedom to surpass itself, was made possible only by the support it found in human nature, developed by the immensity of its past. It was history which, by its progression,

made the basic relation between mind and nature intelligible. For nature never supported mind except through the mediation of time. If that were the case, an education which believed it could take psychology as its basis was symptomatic of the crisis into which we were plunged by anarchical speculation. The knowledge of childhood was not psychological, but historical. So a positivist education was an education based not on the bogus sciences, the latest superstitions of modernity, but on the experience and memory of humanity.

When Comte deprecates 'the deplorable obsession with psychology',[12] he therefore draws attention to the fact that our knowledge is not psychological, but historical. He means in particular that, as the purpose of education is to enable human beings to attain humanity, it is not a question of coming down to the child's level, but of raising the child to the dignity of a person. Subjectivity should not be abandoned to its anarchical drives; the first concern must be to 'model the inside on the outside'. Here we are reminded of Clotilde de Vaux's famous formula: 'Our species more than any other needs duties as a pre-condition for feelings'.[13] It was in this sense that education was liberating. Comte dwelt on the exaggerations of educational psychology, which consisted in giving free rein to the egotistic tendencies to the point of repressing the kindly instincts, which were present in childhood. Education could not therefore avoid restraining inclinations—and indeed very strong inclinations—which, if given free rein, would be detrimental to the development of the mind and the growth of the heart. This kind of suppression did not imply repression of private desires and interests whose reality would thus be ignored. On the contrary, it meant recognizing levels of reality in the human mind and proceeding to order them in accordance with natural laws. For an education which deliberately admitted only the most obvious individual spontaneity would be committing the greatest offence against the child, whose nascent generosity would thus be curbed and placed under an interdict, as it were. Here, the terms accredited by the 'metaphysical' education of absolute drives had to be reversed: repression was at the opposite pole from what it was claimed to be by an education dominated by a psychology which was, in Comte's opinion, as we know, a mere caricature of science. Human beings evolved directly from the biological to the social, from animality to humanity, their true individuality being situated beyond that progression. Education should liberate aspirations hitherto repressed by the imperious urges of a vitality which, from the standpoint of humanity, was not an end in itself. To educate was first of all to protect the weakest inclinations, for they were already on a level with those of a mature person.

So education was not made to perpetuate the state of childhood, either in the individual or in the species. Comte would undoubtedly have found it absurd if he had heard it said that the school was made for the children. If education had a meaning, it was in that it enabled the child, not only to become an adult in mind and feelings, but above all to meditate on the childhood of humanity and its development, which, in fact, was the history of a liberation.

Authority versus power

Only positivist education, based on the philosophical knowledge of humanity, could meet with unanimous free consent. Education was a universal bond. Through it, individuals came to resemble and to know one another, not only in space, but more essentially in time. Through it, too, the great Western republic was already universal, according to the theory whereby all the various situations of humanity were part of the same fundamental evolution. So nothing could be further removed from Comte's thought than the present-day cult of differences, which he would most certainly have perceived as an anti-historical prejudice and a regressive phenomenon. Positivist education made everyone participate in the same history, which was a history of liberation. Philosophy was seen essentially as emancipating.

It will be recalled that the interval of modernity, which Comte did not imagine as interminable, and whose end he sometimes even believed he could predict, succeeded the theological order, in decline as we know, and preceded the truly positive human order. However, this stage of crisis tended by its very nature towards the dissolution of any order whatsoever: it brushed aside any rule, from then on held in advance to be arbitrary, giving free rein to subjective opinions, the manifestation and expression of which placed a curb on what was best in individuals, to the extent of repressing their strictly human vocation.

We can, then, understand why Comte condemned the principle of limitless freedom of conscience,[14] a legacy of the necessary challenge to the old order, but perpetuated beyond all reason. Thus 'there is no freedom of conscience in astronomy': it was enough to have studied. However—as we very well know—education might not always stand up very well to the 'insurrectional habits of modern thought'.[15]

True authority

At the heart of this philosophy of education looms an idea that must not be overlooked. As we rise in the scale of encyclopaedic knowledge, the action of natural forces, as it becomes more complex, becomes more flexible, and when we enter the world of living things, then of the human species and human thought, it is no longer merely forces that act, for their low intensity makes consent necessary for them to be effective.[16] Thus, even in the history of the mind, authority changes in nature: it tends to be less coercive when it is ultimately based on simple acknowledgement of what is true. This inevitable natural weakness of intellectual and moral authority can become a force only through the alliance of vigilance and the generosity at the heart of positive education. Liberated from all transcendence, from the omnipotence of God, spiritual power no longer has any authority except that of the truth. In fact, it is nothing more than humanity having achieved self-knowledge.

Recognizing an authentic spiritual authority presupposed the rejection, as a

result of philosophical progress, of the authoritarian and arbitrary forms of authority—in short, self-appointed authority. In modern times, the worst authority was the acknowledged authority of specialists confined in their narrow spheres of competence. People who knew nothing else but what they believed they knew by virtue of a 'compartmentalized specialization' in fact did not even know what they were supposed to know. Charlatanism thus existed in authority. The performances of a clever mathematician (nowadays we would say a computer), the esotericism of a formal language, the feats of a technique in one of its narrow fields, including in the fine arts, aroused unjustified admiration in the uninitiated, who imagined something they could not do themselves to be the work of superior beings, instead of using their judgement to compare each of those wonders with the achievements of humanity as a whole, in order to appreciate their usefulness and discriminate between the real thing and the insignificant. Positivist education made people free, that is to say, instead of passively undergoing modernity, they were capable of reconciling it with universal history.

Education was not a matter for a special science, but for philosophy

Here we are far from the formative ambitions to which the educational institutions lay claim, and are often confined. A feature of specialized instruction, theoretical or practical, was the ignoring or neglecting of the general principles that might have given it a meaning. Addressing the workers at a people's university, Comte did not offer them further vocational training. He did not talk to them about their trades, their district or their daily routine. He spirited them away from their 'environment' to let them see beyond their world: a result of this teaching was the *Traité philosophique d'astronomie populaire*[17] [Philosophical treatise on popular astronomy]. This approach is obviously the reverse of the 'training courses' people go on today. A genuine school is not designed to provide occupational and further training: it is above all liberating, and the influence Comte's thought was to have on the establishment of the republican school is well known. There are diametrically opposite ways of understanding what is called 'preparation for life', for we might ask 'what is life?'. The purpose Comte proposes for education concerns people who, on emerging from a revolutionary crisis and protracted anarchy, were to inaugurate the positive era which alone could combine order and progress.

It will be seen that the theory of education is not a matter for a special science, but presupposes an all-embracing view, that is, philosophy. It is because humanity is history and that this reminds us that we began with childhood that we can understand childhood and find the paths along which to lead it. The theory of education is no more than the positive representation of the history of humanity.

The cult of memory

The programme for education was therefore first embodied in the philosophy of history that emerged from the 'social dynamics': humanity was built up more through continuity than through solidarity,[18] more over time than over space. Thus education could not merely focus on the present, as was the tendency in a 'society without ancestors'—Comte went so far as to refer to 'the rebellion of the living against all the dead'. On the contrary, education presupposed the cult of memory. One could have no idea of humanity without referring to history. Reverence for the past inclined us to extol our ancestors, that is, those who preceded us. For they were, in a sense, in front of us rather than behind us, and thus showed us the way. No real progress was possible if we lost sight of the historical continuity whereby humanity was constituted. For just as theological dogmas had jeopardized the idea of order, demoralizing criticism, not based on any rule, would eventually jeopardize the idea of progress. It was therefore commemoration and not adherence to the immediate present which should prevail in education.

Humanity's past belonged to everyone. Through memory, our governing faculty, as Pascal[19] had so well understood, the past enabled humanity to fulfil itself by means of an unbroken forward movement. The crime of colonialism, for instance, was not therefore that it underrated cultures foreign to our own, but that it imposed those elements which, in our past and in our present, could jeopardize progress. The combined action of the missionaries and the *littérateurs* had prevented backward populations from passing directly from their initial fetishism to the positive stage, from thus doing without the major crisis with which we were still grappling. 'The crime of the West'[20] therefore consisted in passing on our vices rather than our virtues. The wrong lay not so much in the conquest as in the contamination. Once again it will be seen that nothing in Comte's thought tended towards what is now known as 'cultural pluralism'. Humanity was one and indivisible, whatever the appearances of the moment. However, the West had not fulfilled its educational function, which indeed it had already abandoned in its own case. Thus, Comte invites us to raise our sights to the idea of universal education. Differences were not spatial, nor were they ethnic and cultural, as it is the fashion to say; they were differences in time, that is, they were historical. Then again, it had to be remembered that history did not progress everywhere at the same pace. Spatial, or geographical, diversity was not representative of the persons concerned unless it was related to time and to history. The fetishist was part of the spectrum. Not recognizing the historical dimension of humanity which made us all absolutely alike, and thus forgetting that humanity educated itself, was the most serious offence against the human being. Education did not have to be invented: knowing what we were was enough.

Education should be encyclopaedic

It is not enough just to read Comte; he has to be reread. And then the *Cours de philosophie positive,* which he was to regard later as a mere introduction, takes on its full significance. What matters above all is to rule out the restrictive interpretations suggested nowadays by the word *positivism.* A person had first to learn to adjust to the invariable order of things, but this was so as better to adjust later to human order. Science teaching was therefore just a part of education, and its utility was conditional on its real purpose being kept in sight. If one had first to learn to submit to external order, it was so as to prepare oneself better for the correct appreciation of human order. Science teaching thus served only as a kind of introductory course; it was not an end in itself. That is why Comte's positivism is quite the opposite of a school of scientism which, as we know, would sometimes see fit to cite him as its authority.

In the first place, the 'encyclopaedic' law that governed the classification of the sciences had a supremely educational value. Comte's epistemology is primarily inseparable from what he himself referred to as 'the philosophical history of the sciences', which was part and parcel of the whole history of the human mind. Between ordinary common sense and science there was no break, contrary to the theme of predilection which was to inspire the epistemology of the twentieth century. However, an introduction to science presupposed an epistemological and historical order on which the syllabuses had naturally to be based. One had always to begin at the beginning. Whoever, for reasons of topical interest or technical efficiency, began at the end would have but a smattering of science unrelated to real scientific processes. That explained why science teaching taken to extremes could produce so many pseudo-thinkers. The neglect of encyclopaedic order jeopardized the teaching of science. It was for that reason that the abstract sciences, the only sciences that fitted into an ordered classification, should be taught first. Human intelligence ranged from the abstract to the concrete, from theory to reality. Besides, the concrete sciences could not be fitted into the systematic classification of the sciences, for every concrete science required the contributions of a number of sciences which had first to be studied separately. The path of knowledge, and hence the order of study, thus led from the simple to the complex, from the lower to the higher. For instance, knowledge of living things implied previous knowledge of the inert. In other words, physics and chemistry were a prerequisite for biology. And knowledge of human beings (here Comte invented the term *sociology,* which was subsequently to lose its initial meaning) thus came at the pinnacle of the ascent from the lower to the higher, where a comprehensive approach, which was none other than the approach of philosophy itself, could be freely exercised.

Comte persisted in the idea that true science was made above all to be taught. There was no science outside popular science. However, popular science was not to be understood, as it so often is in our time, as the dissemination of

contents which degenerate as they are dispersed. An encyclopaedia, for example, was not an alphabetical directory from which one could extract a variety of facts and figures at need—still less a data bank. It was the order of thought patterned on the external order, and thus established. It was not with the idea of changing the world or making money that one studied the sciences, but to set one's own thoughts in order. This *polytechnicien* knew that science can be lethal when it is acquired as a means to achieve power or wealth. In short, not only was science teaching far from being the whole of education, as we shall see, but it was not to be conducted with a view to mercenary applications. One of the perversions of our time—Comte forewarned us—is to subordinate intelligibility to technical efficiency.[21] He was already aware that modernity was tempted by a perverse use of the sciences. He had reason to fear, as was to be the case with Jules Verne, that the craze for technologies would drive out the humanities. He went still farther, however: he knew that the fanaticism of the technocrats would finally destroy scientific thought itself.[22]

Education should be general

Comte gave his philosophy of education its full meaning when he inveighed against *'l'usurpation algébrique'*[23] [the despotism of mathematics], or again 'the preponderance of signs over ideas', which tended to give the scientific accolade to those who could skilfully negotiate the secret corridors of what Leibniz in his time had called 'blind thought'. Skill at manipulation led to loss of intellect. The development of science teaching in our own time only goes to justify Comte's apprehension. The confusion of ideas which he condemned is now affecting both the school and society as a whole. Comte especially stresses the catastrophic effects of specialization, which was mainly of use for practical tasks, but which in the field of theory overburdened the mind. Hence his critical exploration of the relations between research and teaching. On the one hand, scientists were appreciated only if they produced practical inventions;[24] on the other hand, the specialization necessary in the world of industry was tending to gain ground in both research and teaching. Thus scientific tasks fell more and more to 'thinkers of little repute',[25] who, for lack of a vocation, were not motivated by anything much except their career prospects. With research thus deprived of strictly scientific thought, the sources of real education were tending to dry up.

So, contrary to the misleading interpretations of positivism, Comte did not hesitate to say that science was a thing of the past. He knew that the great discoveries which humanity could make had for the most part accompanied the historical advent of the positive spirit—something our contemporaries have difficulty in understanding owing to a simplistic idea of progress. What was to follow concerned acquisitions of detail, useful above all for the development or renewal of all manner of technologies. This observation was of decisive importance for the education of the human being of the future. The first rule when setting up education was not to sacrifice the imperishable to what was doomed to pass away and

should be recognized for what it was. Comte unceasingly inveighed against a misguided education obsessed with change, which would remain in the exclusive service of a transitory society. In the first place, all education was general and thus presupposed a philosophical inspiration. Strictly speaking, there could be no such thing as technical education. For this reason, Comte did not attempt to address engineers or technicians, whose minds had become inaccessible. Comte's view, which will appear paradoxical in the extreme to our contemporaries, was that the minds most accessible to philosophical reflection were those of the workers,[26] at least those of his day, who, as yet unspoiled by 'training courses' and spared the cult of profit, were still open to general ideas.

Education and spiritual power

Commemoration, which was to be the keystone of the Religion of Humanity, made it clear that the first human duty was to combat oblivion. So, at the root of education there was history—not psychology, which concealed a principle of confinement. In the first place, the purpose of education was not to perpetuate childhood, except the poetry of infancy, which preserved the better part of the initial fetishism. Education could not, of course, ignore the reality of living organisms, which, situated between the realm of the inert and the realm of the human, underlay the human species without ever constituting it. Education remained blind, however, if it ignored the paths followed by the human species in its early stages. Genuine education was humanity attaining self-awareness while coming to terms with the past which fashioned us.

The idea of spiritual power was bound up with that of an education inspired by a comprehensive approach, that is, philosophy, and thus capable of setting thoughts in order and effecting the intellectual reform appropriate to the modern era. It should not be forgotten—and Comte continually reverted to this—that the most difficult speculations, those that touched on humanity, society and politics, presupposed both encyclopaedic and historical preparation. Otherwise they were idle and baseless and set a bad example, the very opposite of lucidity and wisdom, as was the case when journalists and rhetoricians held sway. The influence of speculation on action created a new duty in that respect. And the difficulty arose precisely from the fact that education and spiritual power were interdependent. Indeed, without such a power no education was possible, for education would then be at the mercy of the temporal and the most obvious interests. And without such education no spiritual authority could be lastingly constituted. Despite the risks, which were only too apparent, we had therefore to rely on the free progress of ideas gradually to reduce the anarchical tendencies resulting from that same progress—a speculative and practical optimism due solely to a supporting philosophical reflection on history.

The term spiritual power might itself be misleading, for in reality that power demanded nothing and did not influence the will. So it could not be a government, for the mind did not obey orders and rejected submission. And to confine

oneself to advising meant having to give up any actual political function, and even wealth.[27] In that sense, education was the opposite of power. Intellectual authority, for instance, existed only if it did not use coercion. Education presupposed an authority which was not the power to constrain or force the will, but merely the capacity to enlighten. Thus real authority was immediately recognized. It was that complete renunciation of power which constituted the real strength of authority. It followed that education could not answer to a temporal power, which in relation to it would inevitably be arbitrary. This is what Comte meant by freedom of education. Its organization in fact required intellectual competence, that is, an authority capable of resisting the arbitrariness of power and, at best, of inspiring it.

Aesthetic education

We know how the finding that science had become a thing of the past was to be understood. Science marked a great turning-point. It remained a mainspring of education. However, only the great beginnings which contributed to the founding of the positive spirit should be called upon. Similarly, if it was considered that the future belonged to art, then art had to be related to its history. Comte spoke of art as he did of science. Both succumbed to the seduction of techniques and narrow specialization. The distinction established between the heart and the mind applied equally to science and to art: there was no genius without inspiration; there was no education without the participation of the feelings.

Nothing brings out more clearly the unity of Comte's thought than his analysis of aesthetic education. Just as science was of value essentially by reason of its approach, and hence the methodical discovery of an external order that liberated the mind, so genuine art was that which in various forms expressed feelings common to humanity, and even revealed those feelings. The whole purpose of education was to enable everyone to discover humanity through its works. However, those works would become a dead letter without initial conviction. Nor was there any education without inspiration. Discovering the genius of Archimedes could move a person to tears.[28] Comte treated art as he treated science, clearly distinguishing its primary purpose, which was to instruct or to charm, from its academic use, even reputedly innovative, which tended to become exclusive in the crisis into which people were plunged by habitually negative criticism. Just as formalism and specialization made us forget that science was thought, so they drove the beauty out of art to produce mere sophisticated curiosities. Music without melody, painting, sculpture or architecture without drawing, were perversions of art, which thus departed from humanity.

Basically, education was learning, that is, the acquisition and ordering of knowledge. 'Real scientific genius' could be recognized in the discovery of 'general laws' which served 'directly to explain a host of everyday phenomena in the midst of which' people lived 'without understanding them'.[29] Likewise, in the matter of art, Comte looked forward to the time when a 'regenerated education'

would 'render singing and drawing as familiar as speech and writing'.[30] This *polytechnicien,* who was a regular Italian opera-goer, no doubt held opera to be the most complete of all the arts: it was through singing that humanity first expressed itself and was revealed to itself. He teaches us how to overcome the long divorce between modern thought and feeling, or imagination.[31] It is then understandable that teachers in all subjects are often incapable of teaching, that is, of kindling the passion to know or to create, of arousing the emotion produced by great works of artistic or poetic genius. If a theorem was not approached in the same way as a poem was discovered, education left the heart indifferent and so deprived us of humanity. It was thus fairly clear that science should appeal to the individual from the level of transcendence, as did art, and not the contrary. Thus education had to start with the highest, instead of starting with the lowest, i.e. with that psychology which was quickly forgotten once truth in action and beauty in memorable works was discovered. If pupils were treated as children they would never become adults. And it was neither the intellectual curiosity so dear to the erudite, nor the technical skill so dear to the practicians, that could constitute the basis of education.

We are brought back to the central idea of spiritual power. It is easy to hold forth on the grandiose dream of a new religion, this time without God or superstitions, which would simply bring people together, and establish and maintain the constituent bond of humanity. The detailed description of the future rites may bring a smile to the lips of more than one reader, apart, perhaps, from the central idea of commemoration through which humanity makes a point of remembering itself, of bearing its true nature in mind. And, indeed, humanity should not forget itself. Comte tells us that our repetitive modernity will not experience a revival unless it regains the fundamental inspiration that long presided over its history. Education undoubtedly needs firmly established institutions, but it also needs that conviction which 'the rebellious habits of modern thought'[32] continue, even nowadays, to repress or destroy.

Notes

1. *Système de politique positive,* Preliminary discourse, vol. I, part V, in particular p. 299.
2. *Correspondance générale,* vol. III.
3. 'Considérations sur la science et les savants', *in: Du pouvoir spirituel,* Paris, Le livre de poche, Pluriel, 1978, p. 249.
4. *Cours de philosophie positive,* 1st lesson.
5. *Le catéchisme positiviste,* p. 78-79.
6. See Alain, *Sentiments, passions et signes,* chapter XLVIII.
7. *Le catéchisme positiviste,* 2nd discussion.
8. Ibid.
9. *Cours de philosophie positive,* 45th lesson.
10. *Système de politique positive,* vol. II, chapter IV.
11. *Ibidem,* Preliminary discourse, part 5.

12. *Cours de philosophie positive,* 45th lesson, p. 854.
13. *Le catéchisme positiviste,* 10th discussion.
14. *Système de politique positive,* General appendix, vol. IV, p. 18.
15. See in particular, *Système de politique positive,* Preliminary discourse, part 3.
16. *Du pouvoir spirituel,* p. 7.
17. See the bibliographical references.
18. *Le catéchisme positiviste,* 2nd discussion, p. 78–79.
19. *Discours sur l'esprit positif,* para. 45. See Pascal, *Traité du vide.*
20. See in particular *Le catéchisme positiviste,* part 3.
21. *Le catéchisme positiviste,* part 1: 'La science constitue toujours un simple prolongement de la commune sagesse. Jamais elle ne crée aucune doctrine essentielle.' [Science is always a mere extension of common sense. It never creates any essential doctrine.]
22. Correspondance générale, vol. I, p. 174.
23. The expression is to be found in Vol. I of *La synthèse subjective,* in particular.
24. See notes 21 and 22.
25. *Cours de philosophie positive,* 46th lesson, Hermann II, p. 76-77.
26. *Passim,* in particular *Système,* chapter 14, p. 81 et seq.
27. *Cours de philosophie positive,* the 45th lesson in particular.
28. *Cours de philosophie positive,* 45th lesson, p. 868.
29. *Système de politique positive,* vol. I, Preliminary discourse, part 5.
30. *Le catéchisme positiviste,* p. 177.
31. *Système de politique positive,* Preliminary discourse, vol. I, p. 275.
32. *Système de politique positive,* Preliminary discourse, part 5, p. 275.

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